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FRANCISCI VIETAE FONTENAEENSIS,

DE ÆQVATIONVM RECOGNITIONE ET EMENDATIONE

TRACTATVS DVO.

Quibus nihil in hoc Genere ſimile aut ſecundum, huic æuo hactenus viſum.

VITTORIO EMANUELE BIBLIOTECA IN ROMA

PARISIIS,

Ex Typographia IOANNIS LAQVEHAY, in Monte D. Hilarij, in Area Albretia.

CIↃ IↃC XV.

CVM PRIVILEGIO REGIS.

BIBLIOTECA NAZ. ROMA VITTORIO EMANUELE

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ILLVSTRISSIMO

AC GENEROSISSIMO

VIRO DOMINO SVO,

D. APOLLINI D'ALBRET

BARONI DE MYOSANS, ETC.

PRÆCLARVM *quidem & egregium,* (*VIR ILLVST.*) *maiorum virtute & gloria, longa iam annorum ſerie repetitâ ſplendeſcere: ſpecioſum, Regia ſtirpe, quâ inter mortales altius ire non datur, nobilem ſe gloriari: ſed qui te, quum omni virtute decorum, tum purioris eruditionis cultura inſignem, & genere, qua nobiliſſimis Euro-*

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BIBLIOTECA NAZ. ROMA VITTORIO EMANUELE

pæ familijs non cedit gens Albretia, & ampliore patrimonio, vberioreque cenſu, liberaliori ſorte vobis conceſſis, exutum, alijſque nudatum fortunæ mendacijs, penitius introſpexerit, non tam alienis hiſce, quam tuis, inſitiſque ac propriis bonis, nobilem te iudicarit: & magnos illos auorum tuorum Manes, qui inter primos Chriſtiani nominis proceres virtute & fortuna pari claruere, digna te reputarit prole beatos; quorum bene præclaréque geſta, æternam ſuo ſæculo peperere memoriam. Quantus ſemper fuerit maiorum tuorum ſplendor & gloria, teſtantur etiam hodie Illuſtriſſimi & Maximi nominis in hoc Potentiſſimo Galliarum regno familiæ, quæ inſigniores Nobilitatis ſuæ notas ex contracta vobiſcum affinitate, repetunt. nec anguſtis ſolum Patriæ finibus incluſum familiæ veſtræ genus inclytum, ſed qua patuit orbis Chriſtianus, in ſupremos Principes & Dynaſtas potentiſſimos, longè latéque propagatum. qui plenioris fidei hiſtorias recolunt, Conſtantinopolitanos, Germanoſque Imperatores, Franciæ, Scotiæ, Angliæ, Hiſpaniæ, Cypri, Hieroſolymarum, Hungariæ, Armeniæ, Siciliæ, Bohemiæ Reges, Apuliæ, Antiochiæque Principes, alioſque quos genus, & altos euexit ad ſidera virtus, quorumque nominibus nulla penè vacat pagina, Affinitate vobiſcum iungi æmulâ contentione expetiuiſſe recognoſcunt. Sed ne vetera longius accerſamus, multo iam clarior Albretiorum

*propago, (cuius nomen gentilitium, omenque Auguſtum, familiæ veſtræ adhuc ſuperſtes.) Immortalitate digniſſimi, & incomparabilis Herois Henrici Magni eius nominis quarti, Galliarum & Nauarræ Regis Chriſtianiſſimi rebus geſtis, & admirandâ virtute, toti terrarum orbi innotuit: qui Caroli Albretij, Gallicanæ militiæ Tribuni (Conneſtabilis patrio idiomate) & Ducis Alani cognomento Magni ſtirpe oriundus, pari ſecum indidem abes gradu: quum Alanus Ioannem Nauarræ Regem, filium maiorem, Henrici Magni ex matre, & Stephanum minorem, veſtrum (*VIR NOBILISS.*) ex patre proauum, fratres genuerit. Magna quidem ſunt hæc generis veſtri & ſuperba nomina, quibus non minor ex eo quoque accedit gloriæ cumulus, quod ſuper alios Principes prædicandum, ſemper Fœlicem & Auguſtum Henricum Magnum, ita ab incunabilis inſtituerit & educarit Illuſtriſſimus & piæ memoriæ auus tuus, vt rerum humarum Arbiter, adeoque orbi vniuerſo regendo, omnium qui hactenus extitere digniſsimus, vnanimi hominum & nationum conſenſu ſit habitus. quis nominis Albretij omen fœlix fauſtumque in hoc principe non veneretur, qui inter arma & militares tumultus, ab Illuſtriſsimi nominis & fœliciſsimæ memoriæ auo tuo, ita eſt educatus, vt non minus pacis artibus, rebus ſua virtute compoſitis, quam armis & militari gloria, inteſtinis*

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odijs adhuc miſerè diuexatâ hac republica, alios Principes antecelleret. quoties Vindicem & Conſeruatorem ſuum agnoſcet Gallia Henricum Magnum, toties Illuſtriſſ. nominis & piæ memoriæ auo tuo obſtrictam ſe prædicabit, qui tantarum virtutum capacem animum, ita à vitiorum fæce (in qua propendet ſemper humanum ingenium) expurgauit, vt optimi principis & fortiſſimi ducis nomina, ſibi propria vindicarit: nec certe exiguus hic ad virtutem aditus, nobili ſtirpe procreari, & ima ac ſordida plebe iam inde ab ortu eximi: nam vt educationem liberaliorem taceam, indoles etiam generoſior & magnitudo animi, claro ortis ſanguine ſtatim ſe prodit, cuius quidem (tum ſingularis eruditionis, qua non ſolum in re Mathematica, ſed & vniuerſa Philoſophia, tum Latina, Græca, Hebræaque literatura, gentem etiam ipſam excellis togatam) vel hic ἔνδειγμα non obſcurum dediſti, quum ſubtiliſſimi viri Franciſci Vietæ opus hoc de Recognitione et Emendatione Æquationum, veteribus quidem vt minus cognitum, ita poſteris ſemper admirandum, neſcio quo fato, iam quatuordecim annos ab Autoris obitu tenebris damnatum, ſtatim vt auditum tibi eſt paucorum vſui, in occulto detineri, quicquid ad id emittendum impendij poſtularetur, liberaliter offerendo, omnes ſanctioris huius ſcientiæ ſtudioſos, hoc beneficio tibi obſtrictos voluiſti. quare Apollinem iam

noſtrum (quo te inſignitum nomine,

Haud equidem ſine mente reor, ſine Numine Diuum.)

Jure merito agnoſcimus & veneramur; fatoque quodam perennitatis, hoc humani ingenij θαῦμα, ſæculorum omnium memoria digniſsimum, tuis Auſpicijs iuris publici factum, acceptum tibi ferimus: & quas alij huiuſmodi artificio inſigniores, pro vno fortaſsis aut altero problemate, mactabant Hecatombas, Auguſti tui nominis æternitati, & fœliciſsimis Franciſci Vietæ Autoris huius artis inuentricis Manibus, vouemus: hocque grati animi monumentum perpetuum, liberalitati tuæ, et eius admirandæ ſagacitatis Genio ſacrum, in publicum damus.

VALE. *Lutetiæ Pariſiorum Prid. Id. Sextiles. Ann. M. DC. XV.*

Illuſtriſſ. Nominis tui
Cultor obſeruantiſſimus,

ALEXANDER ANDERSONVS.

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AD

MATHESEOS STVDIOSOS.

RESTITVTAM Mathematicam Analyſin, Præceptori Franciſco Vietæ debetis φιλομαθεῖς, quam ſuis regulis & præceptis ſuo modo concinnatam, in varia digeſſit opuſcula, quorum quidem nomenclaturam Iſagogicis præmiſſam cernitis. Sed quædam ſiue præcepti & immaturo Autoris fato (nobis certè iniquiſſimo) non dum abſoluta, vel perpolita, vel etiam (vt erat Authoris ingenium.) inchoata, (qui quidem pro ſingulari quâ pollebat animi ſagacitate, multa apud ſe præmeditata, debitoque ordine mente digeſta, ſed nondum ſcriptis conſignata, ob grauiora fortaſſis quæ pro Republica incumbebant munia, ſuis nominibus tanquam confecta inſignire ſolebat.) vel alicubi etiamnum latentia, noſtras nondum manus attigêre. Hæc autem, quorum iam vobis copia conceditur, licet exactiſſimam Autoris limam nondum haud dubiè paſſa, à tanto tamen viro etiam vel ruditer procuſa, toti Mathematicorum ſcholæ vtiſſima, & vt nihil in hoc genere ſimile aut ſecundum hactenus viſum, gratiſſima, diutius ab eruditorum hominum conſpectu, in latebris deliteſcere, ſcelus publico dig-

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num odio duxi. Si quid inde capiunt veſtra ſtudia emolumenti, aut animus ex tam variâ & iucundâ ſpeculatione oblectamenti, quas debituri gratias, Iacobo Alelmo Christianiſſimi Galliarum & Nauarræ Regis LVDOVICI XIII. Architectonicæ militaris ἀρχιμηχανικῷ digniſſimo, viro in hoc ſtudiorum genere apprime verſato, omnes perſoluite: qui quidem Vietæ aduerſaria liberrimè mecum communicauit, ex quibus hæc ſunt deprompta, cuiq; me hoc nomine, vobiſcum arctè obſtrictum profiteor. nec dubito, niſi grauioribus pro principe & patria diſtractus eſſet curis, quibus ſumma cum laude perfungitur, quin & alia eiuſdem viri æternum victura monumenta, in lucem proferret. non eſt quod meum ego hic inſinuam laborem, qui tamen nonnullus, in recenſendo quo vſus ſum exemplari, non paucis in locis deprauato, quibuſdam etiam mutilo: ſingulæ namque æquationes, tum exemplorum notæ epilogiſticæ, nouo ſubijciendæ erant examini, quô quidem pluribus quibus ſparſim ſcatebant mendis, ſunt expurgatæ. quæ vero in textu deeſſe viſa, quâ potui fide pro ſenſus exigentia à me reſtituta ſunt, (ne hęc fortaſſis vt nunc mos inualuit, quaſi præter veritatem dicta, ad faſtum & iactantiam potius trahantur, quam ſtudium in vos meum, quoties de publico bene merendi conceditur occaſio, interpretentur.) iudicio veſtro hic ſubijcientur: cui errata quoque ſubmittimus Typographica, quæ quidem non pauca irrepſere. quamuis in hoc quoque opere dirigendo, non leue mihi impoſitum fuerit negotium. Si quid denique hic parum exactum, aut ſeueriore ipſius Autoris examine minus fortaſſis probatum, pręproperum & pręceps eiuſdem fatum, aut etiam noſtram culpate in vos animi propenſionem, qui ita hęc in veſtrum vſum euulgare maluimus, quàm tot, tamque egregia diuini ingenij monumenta, hoc ſolo opere contenta, noſtris ſcrinijs in priuatum noſtrum bonum recondita, iacere.

VALETE.

Quæ nos ad plenam Autoris mentem in exemplari deeſſe viſa ſuppleuimus, aut a Typographis & Anagnoſtis commiſſa ſiue omiſſa animaduertimus, hæc ferè ſunt.

PAgina 8. in notis Arithmeticis Theor. 2. cap. 5. pro 89. N. lege 8 *Q*.
Pag. 15. lin. 11. hæc omiſſa poſt vocem *ducitur* interſerantur, *quadraticè, cubicè, & vlterius Climacticè. Deſcenſus contrà, quum diuiditur ſubq.* &c.

Pag. 16. lin. 9. pro *ſimplices*, lege *ſimpliciores*.

Pag. 21. in fine, tranſpoſitæ adfectionum notæ ſic reſtituantur,
— Ec. + B in Eq. 3. — Bq. in E 3. + Bc.

Pag. 23. in vltima æquatione Theor. 2. Cap. 11. trãſpoſitas quoque adfectionum homogenei comparationis notas, ſic reponito, *Z ſ. — Dp in B — Bc.*

Pag. 24. The. 4. Cap. 11. pro vltima adfectione cubi abs E — B. id eſt Bc. irrepſit *Dp*.

Ibidem. in vltima eiuſdem Theor. æquatione, in homogeneo ſub gradu, pro *Bc.* repone *Dpl.*

Ibidem. ſub finem. priori ſolidi adfectionis termino, addatur negationis nota —

Pag. 25. eiuſdem Theo. 5. homogeneum comparationis ſic reſtituatur.
Z ſ. + Dp in B — Bc.

Pag. 26. lin. 2. addatur ipſi *Ec.* negationis nota —

Pag. 33. in prima æquatione Theor. 5. pro *Dpp.* lege *Dp*.

Pag. 38. lin. vltima. pro 21 *N*. lege 21 *Q*.

Pag. 44. ſub initium pro, *hiſce omnibus per A luſis* lege *per A — E.*

Pag. 46. ſub finem, pro *A poteſtate in E grad.* lege *E poteſt. in A grad.*

Pag. 58. ad prop. 14. Cap. 18. deerant in Exemplari hæc. *& Z. plano-ſolidum quod fit à B ſolido minus cubo a duabus primis, in quadratum compoſita ex prima & ſecunda.*

Pag. 60. ad prop. 4. Cap. 19. deerant in exemplari noſtro, *differentia eſt B, & fit Z. plano-planum ex ductu vtriuſuis extremæ, in cubum differentiæ reliquarum, alternè ſumptarum.*

Pag 67 lin. 30. lege *plus B in A bis.*

Pag. 80. æquatione vltimâ. pro *Dq — Dp.* lege *Dq — Bp.*

Pag. 81. poſt hæc verba *Conſequenter oportet* &c. pro *omnia ducantur per.* &c. lege *omnia diuidantur.*

Pag. 82. ſub initium, pro *B in Eq. — Acub.* lege *B in Eq. — Ecub.*

Pag. 84. lin. *vtraque pars* &c. pro *Aqq. — A in Ecub.* lege *Eqq.* &c.

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Pag 85. Theor. 5. poſt verba, *quoniam enim* &c. *æquatur Z ſolido*. ſubiunge relicta. *et rurſus B planum in D, — D cub. æquatur Z ſolido.*

Pag. 89. pro 1 *C* + 3 *Q. æquabitur* 130. lege 1 *C.* &c. *æquabitur* 1300.

Pag. 90. ſub linea *diuidantur omnia.* reſtituatur fractio $\frac{8}{4}$ *N*.

Pag. 98. Theor. 2. ad probl. 2. hæc verba exemplaris, *ex applicatione quadrati aggregati laterum, minus triplo rectangulo ſub lateribus, ad rectangulum ſub lateribus.* ita mutauimus *ex applicatione cubi a D plano, ad differentiam quadruplam quadratorum a D & B.*

Pag. 99. lin. 15. pro *fit 1 N* 16. lege. *fit 1 N 6.*

Ibidem. Theor. 3. proximè ſequenti, hæc prorſus deerant in exemplari. *& fit Z plano-planum ex applicatione cubi à D plano, ad aggregatum quadruplum quadratorum a D & B.*

Pag. 105. linea vltima. pro *G plano 4.* lege *G pl.*

Pag. 112. inter lineas *quoniam* & *igitur*. in exemplis Arithmeticis Conſectarij interſeratur. & 378 — 351 *eſt* 27, *cubus a latere* 3.

Pag. 121. ſub linea *Ductis omnibus.* in ſecunda æqualitatis parte lege + *Dp in B* — *B cub.* 2.

Pag. 129. prop. 3. prioris æquationis partis fini adijciatur — *Aqq.*

Ibidem in notis Arithmeticis pro 16 *C.* lege 10 *C.*

De his aliiſque penes lectorem iudicium eſto.

FRANCISCI
VIETAE FONTENAEENSIS
DE RECOGNITIONE AEQVATIONVM TRACTATVS.

De dignoſcenda æquationum conſtitutione ex Zeteſi, Plaſmate, & Syncriſi.

CAPVT. I.

GENERALEM & generaliter traditam de numeroſâ poteſtatum reſolutione doctrinam, informat & perficit tractatus de recognitione æquationum: præparatione enim indigent æquationes ſæpe-numerô, antequam fœliciter explicentur: ac præſertim quum poteſtates de homogeneis magnitudinibus negantur; vel itâ mixtim homogeneæ magnitudines de poteſtatibus negantur & affirmantur vt affectiones negatæ adfirmatis præpolleant, ac deinque quotquot æquationes fractis numeris, vel Aſymmetris exhibentur.

In Geometricis quidem, accidens fractionis vel Aſymmetriæ non ſolet æquationibus offi cere, quò minus εὐμηχανῶς explicētur, ſicuti neque vitium negationis: eſt enim certum ſemper ſubiectum ſub quo operetur Geometra: at obeſt πολυπάθεια, & quô elatior eſt poteſtas, affectioniſque gradus, eo maior ſe prodit in explicando problemate ἄῤῥησις ἢ ἀλογία.

Ecquid vero æquationis quæ propoſita eſt agnitâ conſtitutione

non tentabit Analyſta quô ſaxa & ſcopulos refugiat ? num gnarus Anatomices inuertet, deprimet, attollet, & vndique operabitur ſecurè, nouâ quum res poſtulauerit ſuſceptâ Zeteſi, ſub alio quam qui proponebatur termino, ad propoſitam tamen habenti datam differentiam vel rationem.

Omnino æquationum origo, & prima conſtitutio ſcitu digna eſt, nullâque non ſolertiâ ab Analyſta capeſcenda & adſequenda, quò ſibi pateat ad eam reductionis via.

Aequalitatum conſtitutio, potiſſimum deprehenditur, Zeteſi, Plaſmate, & Syncriſi.

De Zeteſi.

CAPVT. II.

Zeteſin non inſtituet temere neque ἀτέχνως Analyſta, quum pura è puris, affecta ex affectis proficiſci dictet ratio: & ideo ad deprehendendum æquationum ad poteſtates puras pertinentium conditionē, is ex datis duobus lateribus poteſtates inquiret:

Ad æquationes adfectarum ſimpliciter poteſtatum, inueſtigabit ex datâ differentiâ vel ſummâ laterum, vel ſuorum graduum, vna cum dato ſub iis aut eorum gradibus paribus, vel diſparibus facto, alterutrum ex lateribus.

Ad æquationes denique multipliciter affectarum, affectionem affectioni adiunget, ſub diuerſis laterum parodicis gradibus.

Aut etiam non ſe adiget vni quæſtionum formulæ, ſed ſe ſub Zeteticis exercebit, ſub quocunque efficto vel propoſito Themate.

Quæ Geometrico operi aptaturus, recordabitur latera ad extremas lineas rectas in ſerie continuè proportionalium referri, facta vero ſub lateribus aut laterum paribus vel diſparibus gradibus, ad aliquarum e mediis poteſtates.

Et quum incidet in æquationem mechanico ſuo benè obuiam, de illâ condet Theorema ſimile, cuiuſcunque æqualitatis Syſtaticum & Exegeticum.

Aequationem æquationi ſimilem regulariter dicimus, quum vtrobique par eſt Poteſtas, ſeu æque alta, ipſaque affecta afficienſue, ſub pari gradu, & eadem affectionis nota.

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Quod ſi aliud quiddam præterea requiritur ſpeciale & conditionarium, ſimilitudo Anomala eſt.

Conſtitutiua Aequationum Quadraticarum ex Zeteticis.

CAPVT. III.

AFfectorum ſanè quæ adæquantur quadratorum conſtitutio, ex Zeteticis dignoſcitur probè : ſunt autem Aequationum huiuſmodi tres ſpecies: Καταφατική, Αποφατική, Αμφίβολος.

Tribus itaque Theorematis de earum conſtitutione, Analyſta ita poterit ratiocinari.

ΚΑΤΑΦΑΤΙΚΗΣ.

THEOREMA. I.

SI A quadratum plus B in A, æquetur Z quadrato : ſunt tres proportionales radices, quarum media eſt Z, differentia vero extremarum B, & fit A minor extrema.

Ex zetetico ſi placet:

Datâ mediâ trium proportionalium linearum rectarum, & differentiâ inter extremas, inuenire minorem extremam.

Quæ enim in lineis rectis locum habent comparationes, eaſdem ad Radices quaſcunque, ſimplices, planas, ſolidaſue, aut vlterioris ordinis homogenas, poſſe aptari, diſſeruit Campanus, in libro de proportione.

Sit igitur data Z media trium proportionalium, B differẽtia extremarum, & oporteat inuenire minorem extremam.

Eſto illa A. maior igitur erit A plus B. ducatur minor in maiorem, fit A quadratum plus B in A. Et vero quando ſunt proportionales, quod fit ſub extremis æquatur mediæ quadrato, igitur A quadratum plus B in A, æquale eſt Z quadrato: id autem ipſum eſt quod ordinatur.

1. Q. + 10. N. *æquatur* 144. *Fit* L. 144. *media inter extremas differentes per* 10. & *fit.* 1N. *minor extrema ex ſerie trium proportionalium.* 8. 12. 18.

ΑΠΟΦΑΤΙΚΗΣ.

THEOREMA II.

Si Aquadratum minus B in A, æquetur Z quadrato: ſunt tres proportionales, quarum media eſt Z, differentia vero extremarum eſt B. & ſit A maior extrema.

Ex zetetico.

Data media trium proportionalium, & differentia inter extremas inuenire maiorem extremam.

Eſto illa A, minor igitur erit A minus B: ducatur maior in minorem, fit A quadratum minus B in A, æquale Z quadrato. Id autem ipſum eſt quod ordinatur.

1. Q — 10 N, æquatur 144. eſt l. 144 media inter extremas differentes per 10. & fit 1 N. maior extrema ex ſerie trium proportionalium 8. 12. 18.

ΑΜΦΙΒΟΛΟΥ.

THEOREMA III.

Si B in A minus A quadrato, æquetur Z quadrato: ſunt tres proportionales, quarum media eſt Z, aggregatum extremarum B, & ſit A maior, minorue extrema.

Ex Zetetico.

Data media trium proportionalium & aggregato extremarum, inuenire, alterutram extremam.

Sit enim data Z media, aggregatum vero extremarum B, oportet inuenire minorem extremam. eſto illa A, maior igitur erit B, minus A, quare B in A minus A quadrato, æquabitur Z quadrato.

Sed ſit A maior extremarum, erit B minus A, minor extremarum: itaque rurſus B in A, minus A quadrato, æquabitur Z quadrato. Vnde A ſiue de minore extremarum, ſiue de maiore poteſt enunciari.

26 N. — 1. Q. æquetur 144. eſt L. 144. media inter extremas quarũ aggregatum eſt 26. & fit 1 N. minor, maiorue extrema ex ſerie trium proportionalium 8. 12. 18.

RVRSVS

Rurſus Ex Zeteticis.

Conſtitutiua Æquationum Cubicarum, ac primum earum in quibus Affectiones exiſtunt ſub latere.

CAPVT IIII.

ÆQVATIONVM quoque cubicarum affectionibus ſub latere obuolutarum conſtitutio ex Zeteticis, ſcitu digna eſt quo pertinent tria quæ ſequuntur Theoremata.

ΚΑΤΑΦΑΤΙΚΗΣ.

THEOREMA. I.

Si Acubus plus B quadrato in A, æquetur B quadrato in Z. ſunt quatuor continuè proportionales, quarum prima maior minorue inter extremas eſt B, aggregatum vero ſecundæ & quartæ eſt Z. & fit A ſecunda.

Ex Zetetico.

Data prima, & aggregato ſecundæ & quartæ in ſerie quatuor cõtinuè proportionalium, inuenire ſecundam.

Eſto ſecunda A, quarta igitur erit Z minus A: ſolido autem ſub primæ quadrato & quartâ, æquatur cubus è ſecunda: quum ſit vt quadratum primæ, ad quadratum ſecundæ, ita ſecunda ad quartam. itaque A cubus, æquabitur B quadrato in Z, minus B quadrato in A, & per Antitheſin, A cubus plus B quadrato in A, æquabitur B quadrato in Z. vt eſt ordinatum.

Si 1. C. + 64 N. æquetur 2496. Sunt quatuor cõtinue proportionales, quarum prima minor inter extremas eſt L. 64. id eſt 8. aggregatum vero ſecundæ & quartæ $\frac{2496}{64}$ *id eſt 39. et fit 1 N. ſecunda ex ſerie proportionalium 8. 12. 18. 27.*

Et ſi 1. C. + 729 N. æquatur 18954: prima maior inter extremas eſt l. 729. id eſt 27. aggregatum ſecundæ & quartæ $\frac{18954}{729}$ *id eſt 26. et fit 1 N. ſecunda ex eadem ſerie.*

ΑΠΟΦΑΤΙΚΗΣ.

THEOREMA II.

Si A cubus minus B quadrato in A, æquetur B quadrato in D. ſunt quatuor continuè proportionales quarum prima minor inter extremas eſt B, differentia ſecundæ & quartæ D, & fit A ſecunda.

Ex Zetetico.

Datâ primâ minore inter extremas, & differentiâ ſecundæ & quartæ in ſerie quatuor continuè proportionalium, inuenire ſecundam.

Sit data B prima minor inter extremas, differentia vero ſecundæ & quartæ D: oportet inuenire continuè proportionales. Sit ſecunda A quarta igitur erit A plus D. ſolido autem ſub quadrato primæ & quartâ, æquatur cubus è ſecundâ; quare A cubus ęquabitur B quadrato in A, plus B quadrato in D. & per Antitheſin, A cubus minus B quadrato in A, æquabitur B quadrato in D. vt eſt ordinatum.

Si 1.C—64 N æquetur 960. ſunt quatuor continuè proportionales, quarum prima eſt L.64. id eſt 8. differentia vero ſecundæ & quartæ $\frac{960}{64}$ *id eſt 15. et fit 1 N. ſecunda ex ſerie proportionalium* 8.12, 18, 27.

ΑΜΦΙΒΟΛΟΥ.

THEOREMA III.

Si B quadratum in A minus A cubo, æquetur B quadrato in D: ſunt quatuor continuè proportionales, quarum prima maior inter extremas eſt B. differentia vero ſecundæ & quartæ D. & fit A ſecunda.

Ex Zetetico.

Datâ primâ maiore inter extremas, & differentiâ ſecundæ & quartæ in ſerie quatuor continuè proportionalium inuenire ſecundam.

Sit data B prima maior inter extremas, differentia vero ſecundæ & quartæ D. oportet inuenire ſecundam. ſit ſecunda A. quarta igitur erit A minus D. ſolido autem ſub quadrato primæ & quartâ, æquatur cubus e ſecundâ: quare A cubus æquabitur B quadrato in A, minus B quadrato in D & per Antitheſin, B quadratum in A, minus A cubo, æquabitur B quadrato in D.

Si 729.N.—1C æquẽtur 7290. ſunt quatuor continuè proportionales quarum prima eſt l.729. differentia vero ſecundæ & quartæ $\frac{7290}{729}$ id eſt 10. & fit 1N. ſecunda ex ſerie proportionalium 27. 18. 12. 8.

Poteſt autem ea ſecunda duplex eſſe, vt in duplici quatuor continuè proportionalium ſerie quæ ſequitur.

L. 59319. 195. L. 24375. 125.
L. 59319. 78. L. 624. 8.

Stante eâdem primâ maiore inter extremas L.59319. & eâdem differentiâ ſecundæ & quartæ 70. ſecunda hic fit 78. illic 195. Sic ſtante eâdem primâ 36. eâdemque differentiâ ſecũdæ & tertiæ 5. proponitur duplex ſeries triũ proportionalium.

36. 6. 1.
36. 30. 25.

Conſtitutiua Aequationum Cubicarum in quibus Affectiones ſunt ſub quadrato.

CAPVT V.

QVæ autem cubicę affectionibus ſub quadrato obuoluuntur æquationes, iiſdem fere terminis conſtant, quibus in affectionibus ſub latere, vt ex Zeteticis ſimiliter clarum fit: quô pertinebunt trina quoque item enuncianda Theoremata.

ΑΠΟΦΑΤΙΚΗΣ.

THEOREMA. I.

SI A cubus minus B in A quadratum, æquetur B in Z quadratum: ſunt quatuor continuè proportionales, quarum prima maior, minorue inter extremas eſt B, aggregatum vero ſecundæ & quartę eſt Z, & fit A aggregatum primæ & tertiæ.

Ex zetetico.

Data prima & aggregato ſecundæ & quartæ in ſerie quatuor continuè proportionalium, inuenire aggregatum primæ & tertiæ.

Sit data prima B, maior minorue inter extremas, aggregatum vero ſecundæ & quartæ Z, in ſerie quatuor continuè proportionalium, oportet inuenire aggregatum primæ & tertiæ.

Sit illud A, tertia igitur erit A minus B, eſt autem vt A ad Z, ita B. ad $\frac{B \text{ in } Z}{A}$ quare $\frac{B \text{ in } Z}{A}$ erit ſecunda, quum ſit vt aggregatum primæ & tertiæ, ad aggregatum ſecundæ & quartæ, ita prima ad ſecundam.

at rectangulum ſub primâ & tertiâ, æquabitur ſecundæ quadrato itaque $\left\{\begin{array}{l}\text{B in A}\\ -\text{B quadrato}\end{array}\right.$ æquabitur $\frac{\text{B quadrato in Z quadratum}}{\text{A quadrato.}}$ omnia multiplicentur per A quadratũ, & diuidantur per B: igitur $\begin{array}{l}\text{A cubus}\\ -\text{B in A quadratum}\end{array}$ æquabitur B in Z quadratum, id autem eſt quod ordinatur.

Si 1 C. — 8 Q æquetur 12168. ſunt quatuor continuè proportionales, quarum prima inter extremas eſt 8. aggregatum vero ſecundæ & quartæ l. $\frac{12168}{8}$ *id eſt 39. & fit 1 N. aggregatum primæ & tertiæ ex ſerie proportionalium 27. 18. 12. 8.*

ΚΑΤΑΦΑΤΙΚΗΣ.

THEOREMA II.

Si A cubus plus B in A quadratum, æquetur B in D quadratum: ſunt quatuor continuè proportionales, quarum prima minor inter extremas eſt B, differentia vero ſecundæ & quartæ eſt D. & fit A differentia primæ & tertiæ.

Ex Zetetico.

Data prima minore inter extremas, & differentia ſecundæ & quartæ in ſerie quatuor continuè proportionalium, inuenire differentiam primæ & tertiæ.

Sit data B prima in ſerie quatuor continuè proportionalium eademque minor inter extremas, differentia vero ſecundæ & quartæ D. oportet inuenire differentiam primæ & tertiæ.

Eſto illa A, tertia igitur erit A plus B: eſt autẽ vt A ad D ita B ad $\frac{\text{B in D}}{\text{A}}$ quare $\frac{\text{B in D}}{\text{A}}$ erit ſecunda: quum ſit vt differentia primæ & tertiæ, ad differentiam ſecundæ & quartæ, ita prima ad ſecundam at rectangulum ſub prima & tertia, æquatur ſecundæ quadrato, itaque $\begin{array}{l}\text{B in A}\\ +\text{B quadrato}\end{array}$ æquabitur $\frac{\text{B quadrato in D quadratum}}{\text{A quadrato.}}$ omnia ducantur in A quadratũ, & diuidantur per B. igitur A cubus, plus B in A quadratum æquabitur B in D quadratum. id autem ipſum eſt quod ordinatur.

Si 1 C + 8 Q æquetur 1800. ſunt quatuor continuè proportionales, quarũ prima inter extremas eſt 8. differentia vero ſecundæ et quartæ l. $\frac{1800}{8}$ *id eſt 15. et fit 1 N. differentia primæ & tertiæ ex ſeriè proportionalium. 8. 12. 18. 27.*

ΑΜφΙΒΟΛΟΥ.

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ΑΜΦΙΒΟΛΟΥ.

THEOREMA III.

Si B in A quadratum, minus A cubo æquetur B in D quadratum: ſunt quatuor continuè proportionales, quarum prima maior inter extremas eſt B, differentia vero ſecundæ & quartę eſt D, & ſit A, differentia primæ & tertiæ.

Ex zetetico.

Datâ primâ maiore inter extremas, & differentia ſecundæ & quartæ in ſerie quatuor continuè proportionalium, inuenire differentiã ſecundæ & tertiæ.

Sit data B prima in ſerie quatuor continuè proportionalium eademque maior inter extremas, differentia vero ſecundæ & quartæ D. oportet inuenire differentiam primæ & tertiæ. Eſto illa A, tertia igitur erit B minus A: erit autem vt A ad D ita B ad $\frac{\text{B in D}}{\text{A}}$, quare $\frac{\text{B in D}}{\text{A}}$ erit ſecunda: quum ſit vt differentia primæ & tertiæ, ad differentiam ſecundę & quartæ, ita prima ad ſecundam: at rectangulum ſub primâ & tertiâ, æquatur ſecundæ quadrato: Itaque $\text{B quadratum} - \text{B in A}$ æquabitur $\frac{\text{B quadrato in D quadratum}}{\text{A quadrato}}$ omnnia ducantur in A quadratum, & per B diuidantur: Igitur $\left.\begin{array}{l}\text{B in A quadratum}\\ -\text{A cubo}\end{array}\right\}$ ęquabitur B in D quadratum: id autem ipſum eſt quod ordinatur.

Si 27 Q — 1 C. æquentur 2700. ſunt quatuor continue proportionales, quarum prima maior inter extremas eſt 27. differentia vero ſecundæ & quartæ l. $\frac{2700}{27}$ id eſt 10. & ſit 1 N. differentia primæ & tertiæ ex ſerie proportionalium. 27. 18. 12. 8.

Alia inſuper Conſtitutio Cuborum ſub latere Affectorum.

Ex Zeteticis.

CAPVT VI.

SED neque omittenda eſt ex zeteticis quoque depromenda ſingularis quædam conſtitutio cubi, qui ſub latere affirmatè, atque etiam eius qui afficitur negatè: quum videlicet (is enim caſus præmouendus eſt.) quadruplus cubus è triente coefficientis plani, cedit ſolidi datæ menſuræ quadrato: quocirca bina iam proferuntur Theoremata.

THEOREMA. I.

Si A cubus plus B plano ter in A, æquetur D ſolido: eſt B planum quod fit ſub lateribus, á quibus qui fiunt cubi, differunt per D ſolidum, & fit A differentia laterum.

Ex Zetetico.

Datâ differẽtiâ cuborũ, & rectãgulo ſub lateribus, inuenire differẽtiã laterũ.

Si 1 C. + 6 N. æquatur. 7. eſt $\frac{6}{3}$ ſeu 2. rectangulum ſub lateribus à quibus cubi differunt per 7. & fit 1 N. differentia laterum: ex hypotheſi laterum 1. & 2.

THEOREMA. II.

Si A cubus minus B plano ter in A æquetur D ſolido, præſtet autem D ſolidi quadratum, quadruplo B plani cubo: eſt B planum, rectangulum ſub lateribus, a quibus qui fiunt cubi componunt D ſolidum, & fit A aggregatum laterum.

Ex Zetetico.

Dato rectangulo ſub lateribus, & aggregato cuborum, inuenire latera.

Si 1 C. — 6. N. æquatur 9. eſt $\frac{6}{3}$ ſeu 2. rectangulum ſub lateribus, a quibus cubi aggregati faciunt 9. & fit 1 N. aggregatum laterum. ex hypotheſi laterum 1. & 2.

Eadem autem Theoremata Geometricè ita concipientur.

ALITER.

PRIMVM THEOREMA.

Si A cubus, plus B plano ter in A æquetur B plano in D. ſunt quatuor continuè proportionales lineæ rectæ, ſub quarum medijs vel extremis fit B planum: differentia vero extremarum eſt D: & fit A differentia mediarum.

Ex Zetetico.

Data differentia extremarum, & rectangulo ſub medijs vel extremis in ſerie quatuor proportionalium, inuenire differentiam mediarum.

Si 1. C + 24 N. æquetur 56. ſunt quatuor cõtinuè proportionales ſub quarũ medijs vel extremis quod fit planum, æquale eſt $\frac{24}{3}$ id eſt 8. differentia vero extremarum eſt $\frac{56}{8}$ id eſt 7. & fit 1 N. differentia mediarum, ex ſerie continuè proportionalium. 1. 2. 4. 8.

ALITER.

SECVNDVM THEOREMA.

Si A cubus minus B plano ter in A, æquetur B plano in D, ſit autem D ſemiſſis quadratum maius B plano: ſunt quatuor

continuè proportionales lineæ rectæ, ſub quarum mediis vel extremis fit B planum, aggregatum vero extremarum eſt D, & fit A aggregatum mediarum.

Ex zetetico.

Dato aggregato extremarum, & rectangulo ſub medijs vel extremis, in ſerie quatuor continuè proportionalium, inuenire aggregatum mediarum.

Si 1 C — 24 N æquatur 72. ſunt quatuor continuè proportionales, ſub quarum extremis vel medijs quod fit planum, æquale eſt $\frac{24}{3}$ *id eſt 8, aggregatum vero extremarum eſt* $\frac{72}{8}$ *id eſt 9. & fit 1 N. aggregatum mediarum ex ſerie continuè proportionalium.* 1. 2. 4. 8.

Quum autem quadruplus cubus è triente coëfficientis plani, præſtat ſolidi datæ menſuræ quadrato: aliam eo caſu ſortitur æqualitas conſtitutionem, ambiguæ ſeu ei quæ negatur inuersè cõmunem: quô pertinet ſequens Theorema.

THEOREMA. III.

Si A cubus minus B plano ter in A, æquetur D ſolido: cedat autem D ſolidi quadratum, quadruplo B plani cubo: B planum ter in E minus E cubo, æquabitur rurſus D ſolido: & enunciatur E de duobus lateribus, à quibus ſingulis quadrata, adiecta rectangulo ſub ipſis lateribus, faciunt B planum.

Quod autem fit ab vno laterum in quadratum reliqui, adiunctum rectangulo: ſeu aliter quod fit abs aggregato laterum in rectangulum, eſt D ſolidum. A vero enunciatur de aggregato illorum laterum.

Ex zetetico.

Dato plano quod conſtat aggregato quadratorum à duobus lateribus, plus rectangulo ſub ijſdem, & dato inſuper ſolido quod fit abs aggregato laterum, in rectangulum, inuenire latera: vel etiam laterum aggregatum.

Si 1 C. — 21. N. æquetur 20. quoniam quadruplus cubus ex 7. maior eſt quam 400. igitur 21 N. — 1 C. æquabitur 20. & ſunt duo latera, à quibus quadrata adiuncta rectangulo à lateribus, faciunt 21. aggregatum autem laterum, ductum in rectangulum eſt 20. & fit 1 N. in æqualitate inuerſe negata, alterutrum e lateribus, maius minuſue; in directe vero negata, aggregatum ipſorum laterum, ex hypotheſi laterum. 1. & 4.

Neque verò in Geometricâ phraſi hic erit magna diſſimilitudo: Enimverò dicet Geometra, B planum eſſe aggregatum quadratorum à tribus proportionalibus lineis rectis, D vero ſolidum quod fit ab aggregato extremarum in mediæ quadratum ſeu ab alterutra extrema in aggregatum quadratorum â reliquis, & fieri A aggregatum extremarum, E vero primam vel tertiam.

Sic in expoſito themate, 21. eſt aggregatum quadratorum à tribus proportionalibus, & ſolidum 20. eſt ab aggregato extremarum in mediæ quadratum, ſeu ab alterutrâ extremâ in quadrata è reliquis ex ſerie proportionalium 1. 2. 4.

At elegantius & præſtantius ex Analyticis Angularium ſectionũ huiuſmodi æqualitatum conſtitutio eruitur, & ad ἐυμηχανίαν accommodatius in hanc formulam.

ALITER.

TERTIVM THEOREMA.

Si A cubus minus B quadrato ter in A, æquetur B quadrato in D: ſit autem B maior D ſemiſſe, B quadratum ter in E, minus E cubo æquabitur B quadrato in D.

Et ſunt duo triangula rectangula æqualis B hypoterinſæ, ita vt angulus acutus ſubtenſus à perpendiculo primi, ſit triplus ad angulũ acutum ſubtenſum à perpendiculo ſecundi; baſis vero dupla primi, eſt D. & ſit A dupla baſis ſecundi. E vero baſis ſimpla ſecundi, contracta, protractaue longitudine eius quæ poteſt quadrato triplum perpendiculi eiuſdem.

1. C. — 300. N. æquetur 432. vel etiam 300 N. — 1. C æquetur 432. ſunt duo triangula rectangula, quorum hypotenuſa communis eſt 10: ita vt angulus acutus primi, à perpendiculo videlicet ſubtenſus, ſit triplus ad acutum ſecundi, à ſuo quoque perpẽdiculo ſubtensũ; baſis autẽ primi dupla, eſt $\frac{432}{100}$ & 1 N in æqualitate directè negatâ eſt baſis dupla ſecundi: in inuerſè vero negatâ, eſt baſis ſimpla ſecundi, plus minuſue eâ quæ poteſt quadratô triplum perpendiculum ſecundi.

Coſtitutâ hypotenuſa communi 10. baſi ſecundi trianguli 9. ſit perpendiculum eiuſdem ſecundi l. 19.

Primi vero hypotenuſa ſtante 10. ſit baſis $2\frac{16}{100}$. itaque quum in eâ hypotheſi dicetur 1. C — 300 N. æquari 432. fiet 1 N. 18. vel quum dicetur 300 N. — 1. C æquari 432. fiet vnus numerus 9 + l. 57. vel 9 — l. 57.

Atque hæc ad aſſequendum ex zeteticis conſtitutiones adæquatarum quæ affectæ ſunt poteſtatum, exempla nunc ſufficiuntor. Etſi enim de ſimpliciter affectis Theoremata tantũ propoſita ſunt, tamen quemadmodum ad multipliciter affectas poſſint trahi, vix ignorabitur, quando præſertim detegetur earum Plaſma, de quo iam ſuccedit dicendi locus. ipſorum enim zeteticorum à quibus antedictæ conſtitutiones depromptæ ſunt, iam patuit proceſſus ſufficiẽter in expoſitis eclogis, hoc poſtremo Theoremate excepto, quod par eſt reiici in peculiarem zeteticorum ad Angulares ſectiones pertinentium ſiluulam.

De Generali Methodô Tranſmutandarum Aequationum.

CAPVT VII.

PLaſmatis ratio pendet è doctrinâ tranſmutandarum æquationum generaliter imprimis proponenda & demonſtranda.

De Tranſmutatione per Alterationem Radicis.

Æquationum tranſmutatio inſtituitur duplici formæ præcautione; aut enim placet alterari Radicem de quâ primum quæritur, aut manere inuariatam.

Qualiſcũque autem alteratio ſit, primæua Radix & noua, datam habent inter ſe differentiam vel rationem, adeo vt vnâ cognitâ, altera non poſſit ignorari.

Ad opus itaque tranſmutationum, alterata Radix quæſititia conceditur alteri quoque inquirendæ eſſe æqualis, atque adeo ex eâ conceſſione nouam induit ſpeciem, ſub quâ æquatio primum poſita dirigitur, & ordinatur noua.

Ac pluribus quidem modis Radix de quâ quæritur poteſt artificioſe alterari, & nouâ ſpecie exhiberi: ac dirigendi ratio ſub ea nouâ ſpecie ipſam quæ primum propoſita eſt æquationem, conſtans eſt & vniformis, per modos illos quoſcunque.

Quæ vt fiant euidentiora proponatur æquatio quæuis de A Radice exponẽda, & ſit Z magnitudo cui adæquetur reliqua; & oporteat propoſitam illam æquationẽ alteratâ radice arte tranſmutare.

Primum igitur A Radix poteſt alterari, & noua ſpecie exhiberi per additionem: vtpote hac conceſſione & argumentatione A + B eſto E. ergo E — B erit A.

Secundô per ſubductionem, vtpote hac conceſſione & argumentatione A — B eſto E. ergo E + B erit A.

Vel etiã iſtâ B — A eſto E, ergo B — E erit A.

Tertiò per multiplicationem: vtpote hac conceſſione & argumentatione B in A eſto E planum. ergo $\frac{\text{E planum}}{\text{B}}$ erit A.

Quarto per diuiſionem, vtpote concedendo & argumentando $\frac{\text{A planum}}{\text{B}}$ eſto E. ergo B in E erit A planum.

Quinto, per Analogiam rationis explicitæ: vt pote concedendo eſſe vt B ad G, ita A ad E, deinde reſoluendo Analogiam, & argumentando, ergo $\frac{\text{B in E}}{\text{G}}$ erit A.

Sexto per analogiam rationis implicitæ: vt pote concedendo eſſe vt A ad B, ita G ad E: deinde reſoluendo Analogiam & argumentando, ergo $\frac{\text{B in G.}}{\text{E}}$ erit A.

Septimo per Parabolicã hypoſtaſin in generibus quibuscunque æquationum : vt pote in quadraticis, concedendo $\begin{matrix}\text{E quadratum} \\ + \text{A in E}\end{matrix}$ æquari D plano, quæ æquatio eſt quadrati affirmate affecti, deinde argumẽtãdo ergo $\frac{\text{D. planum} - \text{E quadrato.}}{\text{E}}$ erit A

Vel concedẽdo $\left.\begin{matrix}\text{E quadratum} \\ - \text{E in A}\end{matrix}\right\}$ æquari D plano: quæ æquatio eſt quadrati negatè affecti: & argumentando ergo $\frac{\text{E quadratum} - \text{D. plano.}}{\text{E}}$ æquabitur A.

Vel denique cõcedendo $\left.\begin{matrix}\text{E in A} \\ - \text{E quadrato}\end{matrix}\right\}$ æquari D plano, quæ æquatio eſt plani ſub latere negati de quadrato: & argumẽtãdo ergo $\left.\frac{\text{E quadratum} + \text{D plano}}{\text{E}}\right\}$ æquabitur A.

Poſtremo per modos compoſitos, & excogitanda ab artifice & tentanda, quæ ſuo fini magis inſeruire coniiciet, figmenta.

Quamcunque autem ſpeciem induit A, æquatio iuxta eam trãſformabitur, & noua de E ordinabitur, ſi quæ de A enunciantur in æquatione propoſita, eadem enuncientur de nouâ quam A induit ſpecie.

Alteretur enim A per additionem, fingendo A plus B eſſe E, vt ſupra, vnde E minus B fit A. quũ igitur par Poteſtas creabitur abs E minus B, quæ proponebatur ab A, & ſimiles quoque parodici gradus, in quos ducantur inuariandæ coëfficientes, ad efficienda eadem affectionum homogenea: omnino facta huiuſmodi æquabuntur propoſito homogeneo. Expurgentur igitur ſecundum artis præcepta, & ita demum æquatio de E ordinetur: Iam tranſmutata erit in nouam æquationem de E, ideſt ipſa A productâ cremẽto B, enunciandam, quod faciendum erat.

Quod in reliquis quibuſcunque alterandi modis, quibus ſemper A valor exprimitur, locum habere conſpicuum eſt.

De Tranſmutatione inuariatâ Radice.

QVod ad ſecundam præcautionem attinet, inuariatâ Radice æquationem tranſmutare, eſt gradum æquationis deprimere vel attollere.

Climacticus autem ſiue adſcenſus, ſiue deſcenſus, regulariter fit vel irregulariter.

Climacticus regularis adſcenſus fit, quum vtraque propoſitæ æquationis pars, diſtributa ordinate, vel ſecus ex Artificis induſtria, ducitur ſub-quadraticè, ſub-cubicè, & depreſſius ſub-climacticè: adſcito nẽpe ad id ſupplemento dato vel inquirendo: omnia autem poſt ductionem, diuiſionemue congruè ordinantur.

Irregularis autem adſcenſus fit, quum omnia quibus propoſita æquatio conſtat, ſingularia homogenea ducuntur in eundem Parodicum gradum, ſiue purum, ſiue affectum à datis congruentibus magnitudinibus, vel etiam eas adficientem: & ducta congruam interpretationem accipiunt, & ordinationem.

Deſcenſus contrà, quum omnia ſingularia illa homogenea applicantur, ſiue ad Radicem quæſititiam, ſiue Radicis gradum Poteſtate inferiorem, vel pure, vel cum adficientibus, adfectiſue datis congeneribus magnitudinibus: & diuiſa, adſcito nempe ad id ſupplemento dato, vel inquirendo, congruam interpretationem accipiunt, & ordinationem.

Demonſtratio vero in quacunque artificioſa tranſmutandi forma ſtatim euidens fit, quoniam quæ æqualia ſunt longitudine, æqualia quoque ſunt ſimili poteſtate, & contra: neque communis diuiſor vel multiplicator æqualitatem immutat vel rationem.

Atque hæc vtà nobis generaliter propoſita ſunt, ita ſpecialibus indigere præceptis, & exemplis concedendum eſt, quæ diffundentur paſſim in conuenientiores locos, prout edenda artificij tranſmutatorij exegerint ſpecimina.

Singularia de Plaſmate.

Capvt VIII.

PLaſma ineſſe æquationibus intelligitur, quum deducuntur Poteſtates affectæ à puris, vel min⁹ affectis æquealtis, aut etiam altiores à depreſſioribus.

Itaque Plaſma poteſt inſtitui per omnes tranſmutationum modos, præter diuiſionem & Climacticum deſcenſum.

Finis Plaſtices ac præcipuus illius vſus eſt, vt agnitæ ęquationes Plaſmaticę, in ſimplices à quibus eductæ ſunt curentur reſolui, ſi quô id liceat impendio.

Quod quũ deprehendet figmẽtarius, adnotabit ſedulô, ac ſuũ tãdem proferet ſymbolum, in artis commendationẽ & illuſtrationẽ.

Omnino & id eſto primum animaduerſione dignum.

Si qua poteſtas affecta eſt, vel afficit ſub ſingulis gradibus ad eam paradocis, Plaſmatica eſt per modum additionis, vel ſubductionis: qnoniam Radix intelligetur adfecta fuiſſe cremẽto, vel decremento deſumpto ex coefficiente ſub gradu altiore, ſecundum Poteſtatis conditionem.

In quadratis videlicet, affecta fuiſſe Radix, ſemiſſe coefficientis ſub latere: In cubis, triente coefficientis ſub quadrato. In quadrato-quadratis, quadrãte coefficiẽtis ſub cubo. In quadrato-cubis quintante coefficientis ſub quadrato-quadrato: & eo cõtinuo progreſſu.

Sic Quadrata quæcunque adfecta, originẽ ſumũt â puris. Cubi affecti ſub quadrato & latere, à cubis affectis ſub latere. quadrato-quadrata affecta ſub cubo, quadrato, & latere, à quadrato-quadratis affectis ſub quadrato vel latere, aut tam ſub quadrato quam ſub latere. & eô deinceps ordine.

Secundũ eſto, ſi qua poteſtas à radice planâ eſt ſolidáue, vel vlterioris ordinis homogeneâ, Plaſmatica eſt per multiplicationis modum, vel implicitæ Analogiæ.

Sic quadrato-quadratũ affectum ſub quadrato, originem ſumit à quadrato affecto ſub latere: quoniam ducta ſunt omnia quibus æquatio conſtat ſingularia homogenea, in quadratum coefficientis ſub latere: vnde Radix effecti quadrato-quadrati intelligitur media proportionalis inter coefficientem ſublateralem, & eam quæ primum proponebatur Radicem.

Sic cubo-cubus adfectus ſub cubo, originem ſumit à quadrato

quoque

quoque adfecto ſub latere, vnde radix effecti cubo-cubi fit ſecunda continuè proportionalium, qualium coefficiens ſublateralis eſt prima, radix vero quæ primum proponebatur earundem quarta.

Sic cubo-cubus adfectus ſub quadrato-quadrato, originem ſumit à cubo affecto ſub quadrato, ductis omnibus quibus æquatio conſtat ſingularibus homogeneis, in cubum coefficientis ſub quadrato: vnde radix effecti cubo-cubi mediâ proportionalis eſt inter coefficientē ſubquadraticā, & eā quæ primū proponebatur.

Tertium eſto, omne quadrato-quadratum affectū ſub parodico ad illud gradu, vno vel pluribus, Plaſmaticum eſt, ſuam videlicet ducēs per Climacticū adſcensū originē, a quadr. affecto ſub latere.

Afficiatur quadrato-quadratū ſub latere: Æquatio affecti quadrati à quo deducta quadrato quadratica eſt, eam paſſa eſt ſuorum quibus conſtat ſingularium homogeneorum diſtributionem, vt quadrati ſymbolum fecerit partem vnam æquationis, planum vero ſub latere vna cum data comparationis magnitudine, alteram: & vtrâque tandem parte ductâ quadratice, interpretationem congruam acceperit homogeneum ſub quadrato, atque adeo omnia fuerint ordinata.

Adficiatur vero quadrato-quadratum, tam ſub quadrato quam latere: æquatio affecti quadrati, à quo deducta quadrato-quadratica eſt, eam paſſa eſt ſuorum quibus conſtat ſingularium homogeneorum diſtributionem, vt quadratica Poteſtas, vna cum Dieſi dati plani comparationis, vel eiuſdem productione, fecerit partem vnam æquationis, planum vero ſub latere, vna cum Apotome dati plani comparationis, vel eodem producto, partem alteram; atque adeo vtrâque parte ductâ quadratice, omnia fuerint ordinata.

Adficiatur denique quadrato-quadratum ſub cubo: æquatio adfecti quadrati, à quo deducta quadrato-quadratica eſt, eam ſuorum quibus conſtat ſingularium homogeneorum paſſa eſt diſtributionem, vt quadrati ſymbolum vna cum plano ſub latere, fecerit partē vnā æquationis, datum vero planum comparationis alteram & vtrâque parte ductâ quadratice, interpretationem congruam acceperint, vt homogenea tam ſub latere quam quadrato, atque adeo omnia fuerint ordinata.

Quartū eſto & poſtremū, cubicas omnes affectiones per Climacticum irregularem adſcenſum, è quadraticis poſſe deduci, verum

in illis ad primæuas non patere reductionis viam, niſi per æquationes Poteſtatum æque-altarum & affectarum: Aequationum tamẽ inde conſtitutarum ſpectata proprietas maximè iuuat.

Sed & inter Anomalas Plaſmaticas, dignæ ſpeciali notâ ſunt eæ quæ pertinent ad homogenea affectionum, quę de poteſtatibus negantur: creantur illæ ſuâ primœuâ origine à puri quadrati Æquatione, inſtituto Plaſmate per additionem vel ſubductionem: nam quum ad Plaſma adſumitur coëfficiens quælibet maior Radice propoſitâ, ſiue ea adfirmetur per hypotheſin, ſiue negetur, ſemper inciditur in æquationem plani ſub latere negati de quadrato. vnde Poriſma.

Quotieſcumque Poteſtas negatur de Affectionis homogeneo, radicem Poteſtatis eſſe ancipitem: quoniam ex ancipite illa quadraticâ reliquæ omnes fluunt & deducuntur, ſaluâ radicis primæuæ Amphibolia.

Deductiua Quadratorum Affectorum à Puris.

CAPVT. IX.

QVæ omnia vt fiant euidentiora, Theorematum aliquot, ad Plaſmata de quibus monuimus pertinentium, iam ſequitur farrago.

THEOREMA. I.

Si A quadratum, æquetur Z plano.
A + B eſto E.

E quadratum minus B in E bis æquabitur. Z plano — B quadrato.

Quoniam enim A quadratum proponitur ęquale Z plano, eſt autem A + B Radici E æqualis, ergo E — B æquabitur A. itaque quadratum abs E — B ęquabitur Z plano: quadratum autem illud conſtat ſingularibus planis.

E quadrato.
— B in E bis.
+ B quadrato.

quare omnibus his ordinatis E quadratum — B in E bis } æquabitur { Z plano — B quadr.

vt eſt enunciatum.

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THEOREMA. II.

Si A quadratum ęquetur Z plano.
A — B eſto E.

$$\left.\begin{matrix} E \text{ quadratum} \\ + B \text{ in } E \text{ bis}\end{matrix}\right\} \text{æquabitur.} \left\{\begin{matrix} Z \text{ plano} \\ - B \text{ quadrato.}\end{matrix}\right.$$

Quoniam enim A quadratum æquatur Z plano, eſt autẽ A — B Radici E æqualis, A — B æquetur E, ergo E + B æquabitur A: itaque quadratum ab E + B æquabitur Z plano: quadratum autem iſtud conſtat ſingularibus planis.

$$\begin{matrix} E \text{ quadrato.} \\ + B \text{ in } E \text{ bis.} \\ + B \text{ quadrato.}\end{matrix}$$

quare omnibꝰ bene ordinatis $\left.\begin{matrix} E \text{ quadratum} \\ + B \text{ in } E \text{ bis}\end{matrix}\right\}$ æquabitur $\left\{\begin{matrix} Z \text{ plano} \\ - B \text{ quadr.}\end{matrix}\right.$ vt eſt enunciatum.

THEOREMA III.

Si A quadratum æquetur Z plano.
B — A vel A + B eſto E.

$$\left.\begin{matrix} B \text{ in } E \text{ bis} \\ - E \text{ quadrato.}\end{matrix}\right\} \text{æquabitur} \left\{\begin{matrix} B \text{ quadrato.} \\ - Z \text{ plano.}\end{matrix}\right.$$

Quoniam enim A quadratum proponitur æquale Z plano, eſt autem B — A Radici E æqualis: igitur B — E æquabitur A. itaq; quadratum abs B — E ęquabitur Z plano: quadratum autem illud conſtat.

$$\begin{matrix} B \text{ quadrato.} \\ - B \text{ in } E \text{ bis.} \\ + E \text{ quadrato.}\end{matrix}$$

quare omnibꝰ bene ordinatis, $\left.\begin{matrix} B \text{ in } E \text{ bis} \\ - E \text{ quadrato.}\end{matrix}\right\}$ æquabitur $\left\{\begin{matrix} B \text{ quadr.} \\ - Z \text{ plano.}\end{matrix}\right.$ vt eſt enunciatum.

Et ſi A + B æquetur E, igitur E — B æquatur A. itaque quadratũ abs E — B æquabitur Z plano: quadratum autem illud conſtat.

$$\begin{matrix} E \text{ quadrato.} \\ - B \text{ in } E \text{ bis.} \\ + B \text{ quadrato.}\end{matrix}$$

quare omnibꝰ bene ordinatis, $\left.\begin{matrix} B \text{ in } E \text{ bis} \\ - E \text{ quadrato.}\end{matrix}\right\}$ æquabitur. $\left\{\begin{matrix} B \text{ quadr.} \\ - Z \text{ plano.}\end{matrix}\right.$ vt eſt quoque enunciatum.

Deductiua Adfectorum aliquot Cuborum ſub Quadrato, à Cubis adfectis ſub Latere.

CAPVT X.

THEOREMA I.

Si A cubus.
— B quadrato ter in A } æquetur Z ſolido.
A — B eſto E.

E cubus
+ B in E quadratũ ter. } æquabitur { Z ſolido.
+ B cubo bis.

Quoniam enim A cubus minus B quadrato ter in A, æquatur Z ſolido: eſt autem A — B radici E æqualis, igitur E + B æquabitur A. quare cubus ab E + B multatus ſolido abs B quadrato ter in E + B, æquabitur Z ſolido.

Cubus autem abs E + B conſtat.

E cubo.
+ B in E quadratum ter.
+ B quadrato in E ter
+ B cubo

Solidum vero Affectionis. — B quadrato ter in E
— B cubo ter.

Quare omnibus benè ordinatis.

E cubus
+ B in E quadratum ter } æquabitur { Z ſolido
+ B cubo bis.

vt eſt enunciatum.

ALITER.

THEOREMA I.

Si A cubus.
— B quadrato ter in A. } æquetur Z ſolido.
A + B eſto E.

E cubus
— B in E quadratum ter. } æquabitur { Z ſolido.
— B cubo bis.

Quoniam enim A cubus minus B quadrato ter in A, æquatur Z ſolido: eſt autem A + B Radici E æqualis, igitur E — B æquabitur A. quare cubus abs E — B multatus ſolido abs B quadrato ter in E — B, æquabitur Z ſolido, cubus autem abs E — B conſtat.

E cubo.
— B in E quadratum ter
+ B quadrato in E ter
— B cubo.

Solidum vero Affectionis. — B quadrato in E ter.
+ B cubo ter.

Quare omnibus bene ordinatis.

E cubus.
— B in E quadratum ter. } æquabitur. { Z ſolido
—B cubo bis.

vt eſt enunciatum.

THEOREMA. II.

Si B quadratum ter in A.
—A cubo. } æquetur Z ſolido.

B — A eſto E.

B in E quadratum ter.
— E cubo. } æquabitur. { B cubo bis.
— Z ſolido.

Quoniam enim B quadratum ter in A minus A cubo, æquatur Z ſolido: eſt autem B — A Radici E æqualis, igitur B — E æquabitur A: quare ſolidum abs B quadrato ter in B — E, minus cubo abs B — E æquabitur Z ſolido: ſolidum autem illud affectum conſtat.

B cubo ter.
— B quadr. ter in E cubus vero negatus de ſolido illo.
— B cubo.
+ B quadr. in E ter.
— E quadr. in B ter.
+ E cubo.

Quare omnibus bene ordinatis.

B in E quadratum ter.
— E cubo } æquabitur. { B cubo bis.
— Z ſolido.

vt eſt enunciatum.

ALITER.

THEOREMA. II.

Si B quadratum ter in A.
— A cubo. } æquetur Z ſolido.

B + A eſto E.

B in E quadratum ter.
— E cubo. } æquabitur. { B cubo bis.
+ Z ſolido.

Quoniam enim B quadratum ter in A minus A cubo, æquatur Z ſolido, eſt autem B + A Radici E æqualis, igitur E — B, æquabitur A: quare ſolidum abs B quadrato ter in E — B, minus cubo abs E — B æquabitur Z ſolido. ſolidū autem illud affectū cōſtat.

B quadr. ter in E.
— B cubo ter. cubus vero negatus de ſolido illo.
E cubo.
+ B in E quad. ter.
+ B quad. ter in E.
— B cubo.

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quare omnibus bene ordinatis.

B in E quadratum ter.
—E cubo. } æquabitur. { B cubo bis.
+ Z ſolido.

vt eſt enunciatum.

Deductiua adfectorum Cuborum tam ſub Quadrato, quam ſub Latere, à Cubis adfectis ſub Latere.

CAP. XI.

THEOREMA. I.

Si A cubus
+D plano in A. } æquetur Z ſolido,
A — B eſto E. E cubus
—B in E quad. ter.
+B quadrato ter. } in E.
+D plano. } æquabitur. { Z ſolido.
+D pl. in B
+B cubo.

Quoniam enim A cubus + D plano in A, proponitur æquari Z ſolido: eſt autem A + B radici E æqualis, igitur E — B æquabitur A. itaque cubus abs E — B, adiunctus ſolido abs D plano in E — B, æquabitur Z ſolido. cubus autem abs E — B conſtat.

E cubo
— B in E quadr. ter.
+ B quadr. in E ter.
— B cubo. Solidum vero affectionis. + D plano in E.
— D plano in B.

Quare omnibus bene ordinatis.

E cubus
—B in E quadr. ter.
+B quadrato ter. } in E.
+D plano. } æquabitur. { Z ſolido.
+D plano in B
+B cubo.

vt eſt enunciatum.

THEOREMA. II.

Si A cubus
+D plano in A } æquetur Z ſolido.
A — B eſto, E.

E cubus
+ B in E quadratum ter.
+B quadrato ter. } in E.
+D plano. } æquabitur. { Z ſolido
— D plano in B.
— B cubo.

Quoniam enim A cubus + D plano in A, æquatur z ſolido: eſt autem A — B Radici E æqualis: igitur E + B, æquabitur A: itaque cubus abs E + B adiunctus ſolido abs D plano in E + B, æquabitur z ſolido. cubus autem abs E + B conſtat.

E cubo
+ B in E quadratum ter.
+ B quadrato in E ter.
+ B cubo.

Solidum vero affectionis. + D plano in E.
+ D plano in B.

quare omnibus bene ordinatis.

E cubus.
+ B in E quadratum ter.
+ B quadrato ter. } in E.
+ D plano. } in E.

} æquabitur. {

D cubo.
+ D plano in B.
— Z ſolido,

vt eſt enunciatum.

THEOREMA. III.

Si A cubus + D plano in A. } æquetur z ſolido.

B — A ęquetur E.

E cubus
— B in E quadratum ;
+ B quadrato ter } in E.
+ D plano. } in E.

} æquabitur. {

D cubo.
+ D plano in B.
— Z ſolido.

Quoniam enim A cubus, + D plano in A, æquatur z ſolido: eſt autem B — A Radici E ęqualis, igitur B — E æquabitur A. itaq; cubus abs B — E adiunctus ſolido abs D plano in B — E, æquabitur z ſolido: Cubus autem abs B — E conſtat.

B cubo.
— E in B quadr. ter.
+ E quadr. in B ter.
— E cubo.

Solidum vero affectionis. + D plano in B.
— D plano in E.

Quare omnibus bene ordinatis.

E cubus.
— B in E quadratum ter.
+ B quadrato ter } in E.
+ D plano. } in E.

} æquabitur. {

B cubo.
+ D plano. in B.
— Z ſolido.

vt eſt enunciatum.

THEOREMA. IIII.

Si A cubus.
— D plano in A. } æquetur z ſolido.
A + B eſto E.

E cubus
— B in E quadratum ter.
+ B quadrato ter. } in E.
— D plano.
} æquabitur. {
Z ſolido.
+ B cubo.
— D plano in B.

Quoniam enim A cubus — D plano in A, æquatur z ſolido, eſt autem A + B Radici E æqualis: igitur E — B æquabitur A. itaque cubus abs E — B multatus ſolido abs D plano in E — B, æquatur z ſolido, cubus autem abs E — B conſtat.

E cubo.
— B in E quadratum ter.
+ B quadrato in E ter.
— D plano.
Solidum vero affectionis.
— D plano in E.
+ D plano in B.

quare omnibus bene ordinatis.

E cubus
— B in E quadratum ter.
+ B quadrato ter } in E.
— B cubo.
} æquabitur. {
Z ſolido.
+ B cubo.
— D plano in B.

vt eſt enunciatum.

THEOREMA. V.

Si A cubus.
— D plano in A. } æquetur z ſolido.
A — B eſto E.

E cubus
+ B in E quadratum ter.
+ B quadrato ter. } in E.
— D plano.
} æquabitur. {
Z ſolido.
+ D plano in B.
— B cubo.

Quoniam enim A cubus — D plano in A, æquatur z ſolido: eſt eſt autē A — B Radici E æqualis: igitur E + B æquabitur A. itaq; cubus abs E + B multatus ſolido abs D plano in E + B, æquabitur z ſolido. Cubus autem abs E + B conſtat.

E cubo.
+ B in E quadratum ter.
+ B quadrato in E ter
— B cubo.
Solidum vero affectionis.
D plano in E.
— D plano in B.

Quare

quare omnibus bene ordinatis.

E cubus.
+ B in E quadratum ter.
+ B quadrato ter. } in E.
— D plano.
} æquabitur. {
Z ſolido.
— D plano in B.
— D cubo.

vt eſt enunciatum.

THEOREMA. VI.

Si A cubus
— D plano in A. } æquetur z ſolido.
B — A eſto E.

E cubus
— B in E quadratum ter
+ B quadrato ter } in E
— D plano.
} æquabitur. {
B cubo.
— D plano in B.
— Z ſolido.

Quoniam enim A cubus, minus D plano in A, æquatur z ſolido: eſtautem B — A Radici E ęqualis, igitur B — E æquabitur A. itaq; cubus abs B — E multatus ſolido abs D plano in B — E, æquabitur z ſolido: Cubus autem abs B — E conſtat.

B cubo
— B quadrato in E ter.
+ B in E quadratum ter.
— E cubo.
Solidum vero affectionis. — D plano in B.
+ D plano in E.

Quare omnibus bene ordinatis.

E cubus.
— B in E quadratum ter.
+ B quadrato ter
— D plano. } in E.
} æquabitur. {
B cubo.
— D plano. in B.
— Z ſolido.

vt eſt enunciatum.

THEOREMA. VII.

Si D planũ in A
— A cubo. } æquetur Z ſolido.
A + B eſto, E.

D planum.
— B quadrato ter. } in E.
+ B in E quadratum ter.
— E cubo
} æquabitur. {
Z ſolido
+ D plano in B.
— B cubo.

Quoniam enim D planum in A minus A cubo, æquatur, Z ſolido: eſt autem A + B radici E æqualis, igitur E — B æquabitur A. itaque ſolidum abs D plano in E — B, multatum E — B

D

cubo, æquatur Z ſolido. Solidũ autem abs D plano in E — B
E cubo
conſtat.

D plano in E.
— D plano in B.
Cubus vero ablatitius.
+ B in E quadr. ter.
— B quadr. in E ter.
+ B cubo.

Quare omnibus bene ordinatis.

D planũ.
+ B quadrato ter. } in E.
— B in E quadr. ter.
— E cubo
} æquabitur. {
Z ſolido.
+ D plano in B
— B cubo.

vt eſt enunciatum.

THEOREMA. VIII.

Si D planũ in A.
— A cubo
{ æquetur Z ſolido,
A — B eſto E.

D planum.
— B quadrato ter. } in E.
— B in E quad. ter.
— E cubo.
} æquabitur. {
Z ſolido.
— D pl. in B
+ B cubo.

Quoniam enim D planum in A minus A cubo, æquatur Z ſolido: eſt autem A — B Radici E æqualis, igitur E plus B æquabitur A. Itaq; ſolidum abs D plano in E + B multatum E + B cubo, æquabitur Z ſolido. ſolidum autem abs D plano in E + B conſtat.

D plano in E
+ D plano in B.
Cubus vero ablatitius.
— E cubo.
— B in E quadr. ter.
— B quadr. in E ter.
— B cubo.

quare omnibus bene ordinatis.

D planum
— B quadratum ter. } in E.
— B in E quadratum ter.
— E cubo.
} æquabitur. {
Z ſolido.
— D plano in B.
+ B cubo.

vt eſt enunciatum.

THEOREMA. IX.

Si D planum in A.
— A cubo.
{ æquetur Z ſolido.
B — A eſto E.

D planũ.
— B quadrato ter. } in E.
+ B in E quadratum ter.
— E cubo
} æquabitur. {
D plano in B.
— B cubo.
— Z ſolido.

Quoniam enim D planum in A, minus A cubo, æquatur Z ſolido: eſt autem B — A Radici E æqualis, igitur B — E æquabitur A: itaque D planum in B — E, minus B — E cubo, æquabitur z ſolido: ſolidum autem abs D plano in B — E conſtat.

D plano in B.
— D plano in E.

Cubus vero ablatitius.

— B cubo.
+ B quadr. in E ter.
— E quadr. in B ter.
+ E cubo.

Quare omnibus bene ordinatis.

D planũ.
— B quadrato ter. } in E.
+ B in E quadratum ter
— E cubo

} æquabitur. {

D plano in B.
— B cubo.
— Z ſolido.

vt eſt enunciatum.

Deductiua Poteſtatum aliquot, Radices planas ſolidaſue habentium, a Poteſtatibus ſimplicium Radicum.

CAP. XII.

THEOREMA I.

Si A quadratum
+ B in A } æquetur Z plano.
B in A eſto E quadratum.

E quadrato quadratum.
+ B quadrato in E quadratum. } æquabitur B quadrato in Z planum.

Quoniã enim A quadratũ + B in A æquatur Z plano: eſt autẽ B in A æquale E quadrato, igitur $\frac{\text{E quadratum}}{\text{B}}$ æquabitur A: itaque quadratum abs $\frac{\text{E quadrato}}{\text{B}}$ adiunctum plano ſub B & $\frac{\text{E quadrato}}{\text{B}}$.

id eſt $\frac{\text{E quadrato-quadratum.}}{\text{B quadrato.}}$
+ E quadrato. } æquabitur z plano.

Omnia ducantur in B quadratum, ergo

E quadrato-quadratum
+ B quadrato in E quadratum. } æquabitur B quadrato in z planum.

vt eſt enunciatum.

Quum autem ipſum E quadratum Radix ſtatuetur plana, hæc erit æquationis enunciatio.

E plani quadratum.
+ B quadrato in E planum. } æquabitur B quadrato in z planum.

THEOREMA. II.

Si A quadratum,
— B in A. } æquetur z plano.

B in A eſto E quadratum, planumue.

E quadrato-quadratum
—B quadrato in E quadratum. } æquabitur B quadrato in z planum.

ALITER.

E plani quadratum.
—B quadrato in E planum. } æquabitur B quadrato in z planum.

Nec diſſimilis eſt ab ea, quæ in antecedente Theoremate, expoſita eſt demonſtratio.

THEOREMA. III.

Si B in A.
— A quadrato } æquetur z plano.

B in A eſto E quadratum, planumue.

B quadratum in E quadratum.
—E quadrato quadrato. } æquabitur B quadrato in z planum.

ALITER.

E quadratum in E planum.
—E plani quadrato. } æquabitur B quadrato in z planum.

Nec diſſimilis eſt ab eâ quæ in primo huius capitis Theoremate, expoſita eſt demonſtratio.

THEOREMA IIII.

Si A quadratum
+ B in A } æquetur Z plano.

B quadratum in A eſto E cubus.

E cubo-cubus
+ B cubo in E cubum. } æquabitur B quadrato-quadr. in z planum.

Quoniam enim A quadratum + B in A, æquatur z plano: eſt autem B quadratum in A æquale E cubo: igitur $\frac{\text{E cubus}}{\text{B quad}}$ æquabitur A: itaque quadratum abs $\frac{\text{E cubo}}{\text{B quad}}$ plus $\frac{\text{B in E cub.}}{\text{B quad.}}$ id eſt $\frac{\text{E cubo cubus.}}{\text{B quad.quad.}}$ plus

$\frac{\text{E cubo}}{\text{B}}$ æquabitur Z plano. omnia ducantur in B quadrato-quadr. ergo

E cubo-cubus.
+ B cubo in E ſolidum } æquabitur B quadrato-quadr. in Z plan.

vt eſt enunciatum.

THEOREMA. V.

Si A quadratum
— B in A. } æquetur Z plano.

B quadratum in A eſto E cubus, ſolidumue.

E cubo cubus
— B cubo in E cubum. } æquabitur B quad. quad. in Z planum.

Nec diſſimilis eſt ab ea quæ in antecedente Theoremate expoſita eſt demonſtratio.

THEOREMA. VI.

Si B in A
— A quadrato. } æquetur Z plano.

B quadratum in A eſto E cubus, ſolidumue,

B cubus in E cubum
— E cubo-cubo. } æquabitur B quadrato-quadr. in z planum.

Aliter.

B cubus in z ſolidum
— E ſolidi quadrato. } æquabitur B quad. quad. in z planum.

Nec diſſimilis eſt ab eâ quæ in quarto huius capitis Theoremate, expoſita eſt demonſtratio.

THEOREMA. VII.

Si A cubus
+ B in A quadratum. } æquetur z ſolido.

B in A eſto E quadratum.

E cubo-cubus.
+ B quad. in E quad. quad. } æquabitur B cubo in z ſolidum.

Quoniam enim A cubus plus B in A quadratum, æquatur z ſolido: eſt autem B in A æquale E quadrato, igitur $\frac{\text{E quadratum}}{\text{B}}$ æquabitur A. quare ex iis quæ proponũtur. $\frac{\text{E cubo-cubus.}}{\text{B cubo.}}$ plus $\frac{\text{E quad. quad.}}{\text{B}}$ æquabitur z ſolido: omnia in B cubum ducuntor.

Ergo E cubo-cubus, plus B quadrato in E quad. quad. æquabitur B cubo in Z ſolidum.

vt eſt enunciatum.

Quum autem ipſum E quadratum ſtatuetur Radix plana, hæc erit enunciatio.

E plani cubus.
+ B quadrato in E plani quadratum. } æquatur B cubo in Z ſolidum.

THEOREMA. VIII.

Si A cubus.
— B in A quadratum. } æquetur Z ſolido.

B in A eſto E quadratum, planumue.

E cubo cubus
— B quadrato in E quad. quad. } æquabitur B cubo in Z ſolidum.

Aliter.

E plani cubus.
— B quadrato in E plani quadratum } æquabitur B cubo in Z ſolidum.

Nec diſſimilis eſt ab eâ quæ in antecedente Theoremate expoſita eſt demonſtratio.

THEOREMA. IX.

Si B in A qu.
— A cubo. } æquetur Z ſolido.

B in A eſto E quadratum, planumue.

B quadratum in E quadrato-quad.
— E cubo-cubo. } æquabitur B cubo in Z ſolidum.

Aliter.

B quadratum in E plani quadratum
— E plani cubo. } æquabitur B cubo in Z ſolidum.

Nec diſſimilis eſt ab eâ quæ in ſeptimo huius capitis Theoremate, expoſita eſt demonſtratio.

Deductiua Affectorum Quadrato-quadratorum ab Affectis Quadratis.

CAPVT. XIII.

De Quadrato-quadratis affectis ſub Latere.

THEOREMA. I.

Si A quadratum
+ B in A } æquetur Z plano.

A quad. quad.
+ B cubo
+ B in Z planum bis. } in A. } æquabitur. { Z plano plano.
+ B quadrato in Z planum.

Quoniam enim A quadratum plus B in A, æquatur Z plano: igitur per Antitheſin, A quadratum æquabitur Z plano — B in A. ergo A quad. quad. æquabitur Z plano-plano, — B in A in Z planum bis + B quadrato in A quadratum. At affectio B quadrati in A quadratum, interpretationem ex propoſitâ æquatione accipiens, valet B quadratum in Z planum — B cubo in A. quare eâ interpretatione adſcitâ, & omnibus bene ordinatis.

A quad. quad.
+ B cubo.
+ B in Z planum bis. } in A } æquabitur. { Z plano-plano.
+ B quadrato in Z planum.

vt eſt enunciatum.

Si 1Q + 8N. *æquetur* 20.
1QQ + 832 N. *æquabitur.* 1680.

THEOREMA. II.

Si A quadratum.
— B in A. } æquetur Z plano.

A quad quad.
— B cubo.
— B in Z planum bis. } in A } æquabitur. { Z plano-plano.
+ B quadrato in Z planum.

Nec diſſimilis eſt ab eâ quæ in antecedente Theoremate expoſita eſt demonſtratio.

Si 1 Q — 8 N. æquetur 20.
1 QQ — 832 N. æquabitur 1680.

THEOREMA. III.

Si B in A
— A quadrato, } æquetur Z plano.

B cubus.
— B in Z planum bis. } in A
— A quadrato-quadrato. } æquabitur. } B quadrato in Z planum. — Z plano-plano.

Nec diſſimilis eſt ab eà quæ in primo huius capitis Theoremate expoſita eſt demonſtratio.

Si 12 N. — 1 Q. æquetur 20.
1248 N. — 1 QQ. æquabitur 2480.

De Quadrato-quadratis affectis ſub Latere, & Quadrato.

THEOREMA. IIII.

Si A quadratum
+ B in A } æquetur. { S plano. + D plano.

A quadrato-quadratum.
— D plano bis. } in A quadratum
— B quadrato. }
+ B in S planum in A bis. } æquabitur. { S plano-plano. — D plano-plano.

Quoniam enim A quadratum plus B in A, æquatur S plano + D plano, igitur per Antitheſin.

A quadratum
— D plano. { æquabitur. } S plano. — B in A.

vtraque pars ita tranſpoſitæ æquationis, ducatur quadraticè, Ergo

A quad. quad.
— D plano in A quad. bis
+ D plano-plano. } æquabitur. { S plano-plano. — B in A in S planum bis. + B quad. in A quadratum.

Et omnibus rite ordinatis.

A quad quad.
— D plano bis } in A quadratũ
— B quadrato. }
+ B in S planum bis in A. } æquabitur. } S plano-plano. — D plano-plano.

vt eſt enunciatum.

Si

Si 1 Q. + 8 N. *æquetur* 20. *compoſito ex* 15. *&* 5.
1 QQ — 74. Q. + 240 N. *æquabitur* 200.

THEOREMA. V.

Si A quadratum — B in A } æquetur { S plano + D plano plano.

A quad. quad.
— D plano bis } in A quadratum
— B quadrato
— B in S planum bis in A } æquabitur { S plano-plano. — D plano-plano.

Nec diſſimilis eſt ab eâ quæ in antecedente capite expoſita eſt demonſtratio.

Si 1 Q — 8 N. *æquatur* 20. *compoſito ex* 15. *&* 5.
1 QQ — 74. Q — 240 N. *æquabitur.* 200.

THEOREMA. VI.

Si B in A. — A quadrato. } æquetur { S plano. + D plano.

B quadratum
— S plano bis } in A quadratum
— B in D planum bis in A.
— A quad. quad. } æquabitur { S plano plano. — D plano-plano.

Nec diſſimilis eſt ab eâ quæ in quarto huius capitis Theoremate, expoſita eſt demonſtratio.

Si 12 N. — 1 Q. *æquetur* 20. *compoſito ex* 15. *&* 5.
114 Q — 120. N. — 1 QQ. *æquabitur* 200.

De ijſdem Aliter.

THEOREMA VII.

Si A quadratum + B in A } æquetur { S plano. — D plano.

A quad. quad.
+ D plano bis
— B quadrato. } in A quadratum.
+ B in S planum bis in A. } æquabitur { S plano-plano. — D plano-plano.

Quoniam enim A quadratum plus B in A, æquatur S plano minus D plano: igitur per Antitheſin.

A quadratum
+ D plano. } æquabitur { S plano.
— B in A.

Vtraque igitur pars ducatur quadratice: ergo

A quadrato-quadratum.
+ D plano in A quadratum bis } æquabitur { S plano-plano.
+ D plano-plano. — B in S planum in A bis
+ B quadrato in A quadratum.

Et omnibus rite ordinatis.

A quadrato-quadratum
+ D plano bis } in A quadratum.
— B quadrato. } æquabitur { S plano-plano
+ B in S planum bis — D plano-plano.

vt eſt enunciatum.

Si 1 Q. + 8 N *æquetur* 20. *differentiæ inter* 40. & 60.
1 QQ + 16 Q + 960 N. *æquabitur* 2000.

THEOREMA. VIII.

Si A quadratum
— B in A } æquetur { S plano.
— D plano.

A quadrato-quadratum.
+ D plano bis. } in A quadratum. } æquabitur. { S plano-plano.
— B quadrato. — D plano-plano.
— B in S planum bis in A.

Nec diſſimilis eſt ab eâ quæ in antecedente Theoremate expoſita eſt demonſtratio.

Si 1 Q — 8 N. *æquetur* 20. *differentiæ inter* 40. & 60.
1 QQ. + 16 Q. — 960 N. *æquabitur* 2000.

THEOREMA. IX.

Si B in A
— A quadrato. } æquetur { S plano.
— D plano.

B in D planum bis in A
+ B quadrato. } in A quadratum. } æquabitur { S plano-plano.
— S plano bis. — D plano-plano.
— A quadrato-quadrato.

Nec diſſimilis eſt ab eâ quæ in ſeptimo huius Theorematis capite expoſita eſt demonſtratio.

Si 12 N. — 1 Q. *æquetur* 20. *differentiæ inter* 40. & 60.
960 N. + 24 Q — 1 Q Q. *æquabitur*. 2000.

De Quadrato-quadratis affectis ſub Cubo.

THEOREMA. X.

Si A quadratum
+ B in A. } æquetur z plano.

A quadr.quadr.
+ B in z planum bis
+ B cubo
} in A cub. } æquab. Z pl.plano. / Z plano.
+ B quad.
/ Z plann.
+ B quadrato.

Quoniam enim A quadratum plus B in A æquatur z plano: igitur per Antitheſin, A quadratum æquabitur Z plano, — B in A, omnia ducantur in A, ergo A cubus æquatur z plano in A, — B in A quadratum: ſeu aliter ex deſignato A quadrati valore.

A cubus æquatur
Z plano in A.
— B in Z planum.
+ B quadrato in A.

Et vtraque parte huius æquationis diuisâ per z planum + B quad.

A cubus.
+ Z plano in B. / z plann
+ B quad. } æquabitur A.

Quod igitur dicitur, A quadratum æquari z plano, — B in A, idipſum ex deſignato A valore ita exprimitur.

A quadratum æquatur.
Z plano-plano.
— B in A cubum. / Z plann.
+ B quad.

Rurſus venio ad primam propoſitam æquationem:

A quad.
+ B in A. } æquari z plano.

Vtraque pars ducitor quadratice: igitur

A quad.quad.
+ B in A cub. bis.
+ B quad. in A quad. } æquabitur z plano-plano.

Iam affectio B quadrati in A quadratum, interpretationem accipito ex poſtremum deſignato A quadrati valore, ipſa erit

B quad. in Z plano-planum.
— B cubo in A cubum. / Z plano.
+ B quad.

Quâ interpretatione adſcitâ, & omnibus rite ordinatis,

$$\text{A quad.quad.} + \frac{\text{B in Z planum bis} + \text{B cubo}}{\text{Z plano} + \text{B quad.}} \text{ in A cub. æquab. } \frac{\text{Z pl.pl.pl.}}{\text{Z plano} + \text{B quad.}}$$

vt eſt ordinatum.

Si 1 Q. + 14 N. *æquatur*. 147.
1 Q Q. + 20. C. *æquabitur* 9261.

THEOREMA. XI.

Si $\left.\begin{matrix}\text{A quadratum} \\ -\text{ B in A.}\end{matrix}\right\}$ æquetur Z plano.

$$\text{A quad.quad.} \frac{-\text{B in Z planum bis} - \text{B cubo}}{\text{Z plano} + \text{B quadrato.}} \text{ in A cub. æquab. } -\frac{\text{Z plano-pl. pl.}}{\text{Z plano} + \text{B quad.}}$$

Nec diſſimilis eſt ab ea quæ in antecedente Theoremate expoſita eſt demonſtratio.

Si 1 Q — 14 N. *æquetur* 147.
1 Q Q — 20 C. *æquabitur* 9261.

THEOREMA. XII.

Si $\left.\begin{matrix}\text{B in A} \\ -\text{ A quadrato,}\end{matrix}\right\}$ æquetur Z plano.

$$\frac{\text{B cubus.} - \text{B in Z planum bis.}}{\text{B quadrato.} - \text{Z plano}} \text{ in A cubum.} - \text{A quadrato-quadrato. æquabitur } \frac{\text{Z plano-plano-plano.}}{\text{B quadrato.} - \text{Z plano.}}$$

Nec diſſimilis eſt ab eâ quæ in decimo huius capitis Theoremate expoſita eſt demonſtratio.

Si 21 N. — 1 Q. *æquetur* 98.
15. C — 1 Q Q. *æquabitur* 2744.

Deductiua affectorum aliquot Cuborum ab affectis Quadratis.

CAPVT. XIIII.

THEOREMA. I.

Si A quadratum, + B in A. } æquetur Z plano.

B quadratum + Z plano } in A. — A cubo. } æquabitur B in Z planum.

Quoniam enim A quadratum + B in A. } æquatur Z plano.

omnia ducantur in A: igitur.

A cubus + B in A quadratum. } æquabitur Z plano in A.

Sed & ex ijs quæ propoſita ſunt, A quadratũ ęquatur { Z plano — B in A.

quare adſcitâ eâ interpretatione in æquatione cubicâ, ad exprimẽdum valorem ſolidi B in A quadratum.

A cubus + B in Z planum — B quadrato in A. } æquabitur Z plano in A.

& omnibus rite ordinatis.

Z planum + B quadrato. } in A — A cubo. } æquabitur B in Z planum.

vt eſt enunciatum.

Si 1 Q. + 8 N. æquetur 20.
84 N. — 1 C. æquabitur 160.

THEOREMA. II.

Si A quadratum — B in A } æquetur Z plano.

A cubus. — B quadrato — Z plano. } in A } æquabitur B in Z planum.

Nec diſſimilis eſt ab ea quæ in antecedente Theoremate expoſita eſt demonſtratio.

Si 1 Q. — 8 N. æquetur 20.
1 C — 84. N. æquabitur 160.

THEOREMA. III.

Si B in A.
— A quadrato } æquetur z plano.

B quadratum
— Z plano } in A
— A cubo } æquabitur B in z planum.

Nec diſſimilis eſt ab eà quæ in primo huius capitis Theoremate, expoſita eſt demonſtratio.

Si 12 N. — 1 Q. æquatur 20.
124 N. — 1 C. æquabitur 240.

THEOREMA IIII.

Si A quadrato.
+ B in A } æquetur B in D.

A cubus.
+ B + D in A quadratum. } æquabitur B in D quadratum.

Quoniam enim A quadratum + B in A, æquatur B in D, omnia ducantur in A, igitur A cubus + B in A quadratum. } æquabitur B in D in A

Sed & ex iis quæ proponũtur (B in D — A quadrato.) / B } æquatur A.

Quare ea interpretatione adſcitâ ad exprimendum valorem ſolidi B in D in A. A cubus + B in A quadratum. } æquabitur B in D quadratum. — D in A quadratum.

& adbibita congrua methateſi

A cubus,
+ B + D in A quad. } æquabitur B in D quadratum.

vt eſt ordinatum.

Si 1 Q. + 16 N. æquatur 80. facto ex 16. in 5.
1 C. + 21. N. æquabitur 400.

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THEOREMA. V.

Si A quadratũ — B in A } æquetur B in D.

B + D in A quadratum. — A cubo } æquabitur B in D quadratum.

Nec diſſimilis eſt ab ea, quæ in antecedente Theoremate, expoſita eſt demonſtratio.

Si 1 Q. — 16 N. *æquetur* 80. *facto ex* 16. *in* 5.

21. Q. — 1 C. *æquabitur* 400.

THEOREMA. VI.

Si B in A — A quadrato. } æquetur B in D.

B — D in A quadratum. — A cubo. } æquabitur B in D quadratum.

Nec diſſimilis eſt ab eâ quæ in quarto huius capitis Theoremate, expoſita eſt demonſtratio.

Si 9 N. — 1 Q. *æquetur* 18. *facto ex* 9. *in* 2.

7 Q — 1 C. *æquabitur* 36.

Ambiguitates Radicum quarum Poteſtates de homogeneis adfectionum in adæquationibus negantur, demonſtratæ.

CAP. XV.

OStendendum eſt quum in æquationibus Poteſtates de homogeneis adfectionum negantur, Radices eſſe Ancipites.

Proponatur differentia inter B & A eſſe æqualis S: & ſit B maior quam S. aut igitur exceſſus eſt penes B, vel penes A.

Primo caſu, B — A æquetur S: itaque B — S fit A

Secundo caſu A — B æquetur S. itaque B + S fit A. quoniam autem primo caſu B — A æquatur S, vtraque pars æqualitatis ducatur quadratice. igitur B quadratum — B in A bis + A quadrato. } æquabitur S quadrato.

& æqualitate ordinata. B in A bis. — A quadrato. } æquale fit { B quadrato. — S quadrato.

Quoniã vero ſecundo caſu, A — B æquatur S, vtraque pars æqualitatis ducatur quadraticè, igitur B quadratum — B in A bis. + A quadrato. æquabitur S quad. & æqualitate ordinata, B in A bis. — A quad. rurſus æquale fit. B q. — S q.

Vtroque igitur caſu in eandem inciditur æquationis formulam: eſt autem Radix duplex: ipſa vero formula æquationis eſt, vt planum ſub latere, adficiatur multâ quadrati.

Sit B 6. S. 4. A. 1 N.

1 2 N. — 1 Q. *æquatur* 20. *& fit* 1 N. 6 — 4. *vel* 6 + 4

Quam Amphiboliam in omnibus ſimilibus æquationibus locum habere ſatis intelligitur, vt ſi proponatur D in A — A quadrato. æquari Z plano: ipſa D, dicetur eſſe B bis, & Z planum eſſe S quadratum minus B quadrato.

Porro, per ſextum Theorema antecedentis Capitis, apparet ab ea quadrati ambigui æquatione, deriuari cubos negatos de ſolidis ſub Parodico gradu: & per tertium, ſextum, & nonum, Capitis duodecimi, ab eadem deriuari quadrato-quadrata negata de plano-planis ſub Parodico gradu, quod ad vlterioris ordinis æquationes poſſe extendi ſatis fit manifeſtum.

De Syncriſi.

CAP. XVI.

Syncriſis eſt duarum æquationum correlatarum mutua inter ſe ad deprehendendum earum conſtitutionem collatio.

Duæ autem æquationes correlatæ intelliguntur, quum ambæ ſimiles ſunt, & prætereà iiſdem datis magnitudinibus conſtant, ſiue adfectionum Parabolis, ſiue adfectionum homogeneis. Radices tamen ideo diuerſæ ſunt, quoniam vel ipſæ æquationum formulæ de duabus pluribuſue Radicibus, ex ſui cõſtitutione ſunt explicabiles, vel in ijs diuerſa eſt adfectionum qualitas, ſeu nota.

Ac de ſimplicioribus quidem correlatis ſufficiet doctrina, id eſt vna tantum adfectione obuolutis, vt pote quarum vi, de ijs quæ paſſionibus

ſionibus abundant, Plaſtices peritus ratiocinabitur ſecure.

Correlatarum igitur ſimpliciorum ęquationum, tres ſunt differentiæ.

Prima eſt Ancipitum, in quarum vtraque Poteſtas negatur de homogenea adfectionis.

Altera eſt Contradicentium, in quarum vna Poteſtas adficitur adfirmate, in altera negate.

Tertia eſt Inuerſarum, in quarum vna, Poteſtas adficitur multâ homogeneæ adfectionis, in altera e contrâ, homogenea adfectionis multatur Poteſtate.

Siue autem Ancipitum, ſiue Contradicentium, ſiue Inuerſarum ęquationum conſtitutio, Syncriſi cognoſcetur probè: Ineundæ ſiquidem Syncriſeos hæc eſt ratio. quoniam enim quæ vni æquātur inter ſe æqualia ſunt: proponuntur autem Poteſtates duæ adficientes, adfectæ uæ, eidem datæ omnino magnitudini homogeneæ comparari, ipſum autem adfectum, adficiēſue homogeneum, vtriuis poteſtati immixtum, fieri ſub Parodicis ijſdem gradibus, eademque parabola: ergo Poteſtas Radicis vna cum homogeneo ſub ſuo gradu Parodiço, æquabitur Poteſtati alterius vna cum homogeneo ſub ſuo quoque gradu Parodico: & tranſlatis ad vnam ita conſtitutæ quoque ęquationis partem Poteſtatibus, ad alteram vero, ijs quæ ſub gradibus fiunt adfectionum homogeneis, erit rurſum æqualitas. vnde quum differētia aut aggregatum Poteſtatum, applicabitur ad differentiam vel aggregatum laterum, (vter vero terminus, opus indicabit.) faciet oriri magnitudinem Parabolæ æqualem: cuius ideo euidens erit conſtitutio.

Quod ſi poſtea vtrauis poſitarum æquationum, Syncriſi expoſitarum, non iam ſub ipſamet Parabola exhibeatur, ſeu ſub æquo Parabolæ (prout eius conſtitutio agnita fuit.) valore: quod ordinabitur, erit conſequenter dato comparationis homogeneo æquale, cuius ideo conſtitutio non poterit ignorari: quoniam orietur illud ex facti ab aggregato vel differentia Radicum, aut graduum, in planum ſub Radicibus applicatione ad eundem cui antecedens applicatio facta eſt terminum, ſiue differentiam, ſiue aggregatum.

Secundum quæ, proficiſcuntur Parabolæ adfectionum in Ancipitibus, ex applicatione differentiæ Poteſtatum ad differentiam eorum quos metitur Parabola graduum.

Homogenea vero comparationum, ex applicatione differentiæ factorum reciprocè, à Poteſtate vnius Radicis in gradum alterius, ad prædictam cui altera applicatio facta eſt, graduum differentiam. Id autem obiter eſt demonſtrandum.

PROBLEMA. I.

DVarum Ancipitum æquationum conſtitutionem, ex Syncriſi dignoſcere.

Proponatur $\left.\begin{array}{l}\text{B parabola in A gradum.}\\ -\text{A poteſtate.}\end{array}\right\}$ æquari z homogeneæ.
& rurſus eadem $\left.\begin{array}{l}\text{B parabola in E gradum.}\\ -\text{E poteſtate.}\end{array}\right\}$ æquari z homogeneæ.

Vnde par exiſtit gradus & par Poteſtas: oportet ex Syncriſi earũ æquationum conſtitutionem dignoſcere.

Quoniam igitur quæ vni æquantur æqualia ſunt inter ſe: manifeſtum eſt. $\left.\begin{array}{l}\text{B parabolam in A gradum.}\\ -\text{A poteſtate.}\end{array}\right\}$ æquari. $\left\{\begin{array}{l}\text{B parabolæ in E gradum.}\\ -\text{E poteſtate.}\end{array}\right.$

& per Antitheſin, ſtatuẽdo ſi placet A maiorem quam E, quod quidem hic liberũ eſt, propter præſuppoſitã Radicũ ἀμφιβολίαν.
$\left.\begin{array}{l}\text{A poteſtas.}\\ -\text{E poteſtate}\end{array}\right\}$ æquabitur $\left\{\begin{array}{l}\text{B parabolæ in A gradum.}\\ -\text{B parabola in E gradum.}\end{array}\right.$

Et omnib. per A gradũ — E gradu diuiſis $\left.\dfrac{\text{A poteſtas} - \text{E poteſtate.}}{\text{A gradui.} - \text{E gradu.}}\right\}$ æqu. B parab.

Oritur ergo B parabola, ex applicatione differentiæ Poteſtatum ad differentiam graduum: vt eſt conſtitutum.

Porro, quum B parabola in A gradum, minus A Poteſtate, æquetur z homogeneæ, in locum B paraboles ſubrogetur iam agnitus eius valor, & ea ſubrogatione æquatio reformetur: ergo

$\left.\dfrac{\text{A poteſtas in E gradum} - \text{E poteſtate in A gradum}}{\text{A gradui} - \text{E gradu.}}\right\}$ æquabitur z homogeneæ.

Oritur ergo z homogenea ex applicatione differentiæ factorum reciproce, a Poteſtate vnius Radicis in gradum alterius, ad differentiam graduum: vt eſt quoque conſtitutum.

In Specie.

Proponatur B in A − A quadrato. } æquari z plano.

Et rurſus B in E − E quadrato. } æquari z plano.

Oportet ex Syncriſi earum æquationum conſtitutionem dignoſcere: quoniam igitur quæ vni ęquantur æqualia ſunt inter ſe, manifeſtum eſt B in A − A quadrato } æquari { B in E. − E quadrato

Et per Antitheſin, ſtatuendo ſi placet A maiorem quam E, quod quidem hic liberum eſt, propter ſuppoſitã Radicum ἀμφιβολίαν.

A quadratum. − E quadrato } æquabitur { B in A − B in E.

& omnibus per A − E diuiſis.

(A quadratum − E quadrato) / (A − E:) } id eſt A + E, æquabitur B.

Eſt igitur B ſumma duorum de quibus quæritur laterum, oriunda ex applicatione differentiæ quadratorum, ad differentiam laterũ, vt eſt generaliter conſtitutum.

Porro quũ B in A − A quadr. } æquetur Z plano, ſi in locum B, ſubrogetur iam agnitus ipſius valor (A quadratum. − F quadrato.) / (A − E.) ſeu A + E.

(A quadratum in E. − E quadrato in A.) / (A − E) id eſt E in A. æquabitur z plano.

Eſt igitur Z planum, id quod fit ſub duobus de quibus quæritur lateribus, ortum ex differẽtiæ factorum reciproce à quadrato vnius, in Radicem alterius applicatione, ad differentiam laterum: vt eſt quoque generaliter conſtitutum.

Aliud.

Proponatur B planum in A − A cubo } æquari z ſolido.

& rurſus B planum in E − F cubo. } æquari z ſolido.

Oportet ex Syncriſi earum æquationum conſtitutionem dignoſcere: quoniam igitur quæ vni æquantur æqualia ſunt inter ſe, manifeſtum eſt B planum in A − A cubo. } æquari { B plano in E − E cubo.

Et per Antitheſin, ſtatuendo ſi placet A maiorem quam E, quod

quidem hic liberum eſt, propter ſuppoſitam Radicum ἀμφιβολίαν.

$$\left.\begin{array}{l}\text{A cubus.}\\ -\text{E cubo.}\end{array}\right\}\ \text{æquabitur}\ \left\{\begin{array}{l}\text{B plano in A.}\\ -\text{B plano in E.}\end{array}\right.$$

Et omnibus per A diuiſis $\dfrac{\text{A cubus.} - \text{E cubo}}{\text{A} - \text{E}}$ id eſt $\left.\begin{array}{l}\text{A quad.}\\ +\text{E quad.}\\ +\text{A in E.}\end{array}\right\}$ æqua. B pl.

Eſt igitur B planum aggregatum quadratorum a duobus de quibus quæritur lateribus, adiunctum ei quod ſub ijs fit plano: & oritur ex applicatione differentiæ cuborum ad differentiam laterũ: vt eſt generaliter conſtitutum.

Porro quũ $\left.\begin{array}{l}\text{B planum in A}\\ -\text{A cubo}\end{array}\right\}$ æquetur Z ſ. ſi in locũ B pl. ſubrogetur. agnitus eius valor, nempe $\dfrac{\text{A cubus.} - \text{E cubo.}}{\text{A} - \text{E}}$. ſeu $\begin{array}{l}\text{A quadratum.}\\ +\text{E quadrato}\\ +\text{A in E.}\end{array}$

$\dfrac{\text{A cubus in E.} - \text{E cubo in A.}}{\text{A} - \text{E}}$ id eſt $\begin{array}{l}\text{A quadratum in E}\\ +\text{E quadrato in A.}\end{array}$ æquabitur Z ſolido.

Eſt igitur Z ſolidum quod fit ab aggregato laterum, in planum ſub lateribus, & oritur ex differentiæ ipſorum factorum, reciproce à cubo vnius lateris in latus alterius, applicatione ad differentiam ipſorum laterum: vt eſt quoque conſtitutum.

Conſtitutio igitur propoſitarum æquationum tandem ex Syncriſi agnita eſt: vt faciendum erat.

Placeat exhibere formulam æquationis Ancipitis quadraticæ, de Radice F & G, hac minore, illa maiore explicabilis.

dixero $\left.\begin{array}{l}\text{F} + \text{G in A}\\ -\text{A quadrato}\end{array}\right\}$ æquari F in G: & fieri A, F vel G.

Sit F 10. G 2. A 1 N. formula æquationis erit 12 N. — 1 Q. *æquabitur* 20. *explicabilis de* 10. *vel* 2.

Placeat exhibere formulam æquationis cubi negati de homogeneo adfectionis ſub latere, vt ſit Radix de F vel G explicabilis.

dixero $\left.\left.\begin{array}{l}\text{F quadratum}\\ +\text{G quadrato}\\ +\text{F in G.}\end{array}\right\}\text{in A}\ \begin{array}{l}\\ \\ \\ -\text{A cubo.}\end{array}\right\}$ æquari $\left\{\begin{array}{l}\text{F in G quad.}\\ +\text{G in F quad.}\end{array}\right.$ & fieri A, F vel G

Sit F 10 G 2. A 1 N. *formula æquationis erit* 124 N. — 1 C. *æquabitur.* 240. *explicabilis de* 10 *vel* 2.

In Contradicentibus, coëfficientes ſub graduales proficiſcuntur ex applicatione differentiæ Poteſtatum, ad aggregatum eorũ quos ſuſtinent coëfficientes graduum.

Homogenea vero datæ menſuræ, ex applicatione aggregati factorum reciproce, à Poteſtate Radicis vnius in gradum alterius, ad prædictum cui altera applicatio facta eſt graduum aggregatum: id autem quoque obiter eſt demonſtrandum.

PROBLEMA. II.

DVarum contradicentium æquationum conſtitutionem ex Syncriſi agnoſcere.

Proponatur $\left.\begin{matrix}\text{A poteſtas.}\\ +\ \text{B coefficiente in A gradũ}\end{matrix}\right\}$ ęquari z homogeneo.

Et rurſus $\left.\begin{matrix}\text{E poteſtas}\\ -\ \text{eadẽ B coefficiẽte in E gradũ.}\end{matrix}\right\}$ æquari eidẽ Z homogeneo.

Vnde par exiſtit gradus & par Poteſtas: oportet ex Syncriſi earum æquationum conſtitutionem agnoſcere.

Quoniam igitur quæ vni æquantur æqualia ſunt inter ſe, manifeſtum eſt.

$\left.\begin{matrix}\text{A poteſtatem}\\ +\ \text{B coefficiente in A gradum.}\end{matrix}\right\}$ æquari $\left\{\begin{matrix}\text{E poteſtati}\\ -\ \text{B coefficiente in E gradum.}\end{matrix}\right.$

& per Antitheſin $\left.\begin{matrix}\text{E poteſtatem}\\ -\ \text{A poteſtate.}\end{matrix}\right\}$ æquari $\left\{\begin{matrix}\text{B coefficienti in E gradum.}\\ +\ \text{B coefficiente in A gradum.}\end{matrix}\right.$

Itaque omnibus per E gradum + A gradu diuiſis.

$\left.\frac{\begin{matrix}\text{E poteſtas.}\\ -\ \text{A poteſtate.}\end{matrix}}{\begin{matrix}\text{E gradui}\\ +\ \text{A gradu}\end{matrix}}\right\}$ æquabitur B coëfficienti.

Oritur ergo B coefficiens ſubgradualis, ex applicatione differentiæ Poteſtatum, ad aggregatum eorum quos ſuſtinet coefficiẽs graduum: vt eſt conſtitutum.

Porro quum $\left.\begin{matrix}\text{A poteſtas.}\\ +\ \text{B coefficiente in A gradum.}\end{matrix}\right\}$ ęquetur z homogeneo.

In locum B coefficientis, ſubrogetur iam agnitus eius valor, videlicet $\left.\frac{\begin{matrix}\text{E poteſtas}\\ -\ \text{A poteſtate}\end{matrix}}{\begin{matrix}\text{E gradui.}\\ +\ \text{A gradu.}\end{matrix}}\right\}$ & eà ſubrogatione æquatio reformetur.

ergo A poteſtas plus $\left.\frac{\left.\begin{matrix}\text{E poteſtate}\\ -\ \text{A poteſtate}\end{matrix}\right\}\text{ in A gradum.}}{\begin{matrix}\text{E gradui.}\\ +\ \text{A gradu.}\end{matrix}}\right\}$ æqu. z homogeneo.

hoc eſt omnib. rite ordinatis. $\left.\frac{\begin{matrix}\text{A poteſtas in E grad.}\\ +\ \text{E poteſtate in A grad}\end{matrix}}{\begin{matrix}\text{E gradui.}\\ +\ \text{A gradu.}\end{matrix}}\right\}$ æq. z. homogen.

Oritur ergo Z homogenea, ex applicatione aggregati factorum reciprocè a Poteſtate Radicis vnius, in gradũ alterius, ad prædictũ cui altera applicatio facta eſt graduum aggregatum : vt eſt ſecundo loco enunciatum.

In Inuerſis plane negatis, coefficientes ſubgraduales proficiſcũtur ex applicatione aggregati Poteſtatum, ad aggregatum eorum in quos coefficiens eſt graduum.

Homogenea vero datę menſuræ, ex applicatione differentiæ factorum reciprocè à Poteſtate Radicis vnius, in gradum alterius, ad prædictum cui altera applicatio facta eſt graduum in quos coefficiens ſubgradualis eſt, aggregatum.

Idem quoque obiter eſt demonſtrandum.

PROBLEMA. III.

DVarum Inuerſarum æquationum conſtitutionem, ex Syncriſi agnoſcere.

Proponatur { A poteſtas. — B coefficiente in A gradum. } ęquari z homogeneo.

& rurſus eadẽ { B coefficiens in E gradum — E poteſtate. } æquari eidem z homog.

Vnde intelligantur A & E pares gradus, atque adeo pares Poteſtates: Oporteat autem æqualitatum harũ conſtitutionem agnoſcere. Quoniam igitur quę vni ęquantur æqualia ſunt inter ſe, manifeſtum fit ex ijs quæ proponuntur.

{ A poteſtatem — B coefficiente in A gradum. } æquari { B coefficienti in E gradum. — E poteſtate }

& per Antitheſin { A poteſtatem + E poteſtate. } æquari { B coefficienti in A gradum. + B coefficiente in E gradum.

Itaque omnibus per E gradum + A gradu diuiſis.

{ A poteſtas. + E poteſtate / A gradui + E gradu } æquabitur B coëfficienti.

Idipſum autem eſt quod primo loco enunciatur.

Porro quum { A poteſtas. — B coefficiẽte ſubgraduali in A grad. } ęquetur z homog.

In locum B coefficientis, ſubrogetur æquus valor, videlicet.

{ A poteſtas + E poteſtate. / A gradui. + E gradu. } igitur A poteſtas minus { A poteſtate in A gradum. + A poteſtate in E gradum. / A gradui + E gradu.

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æquabitur z homogeneo: hoc eſt omnibus rite ordinatis.

$$\left.\frac{\text{A poteſtas in E gradum.} - \text{E poteſtate in A gradum.}}{\text{A gradui.} + \text{E gradu.}}\right\}\text{æquabitur z homogeneo.}$$

Quod ipſum eſt quod ſecundo loco enunciatur.

In Inuerſis autem quarum vna per adfirmationem adficitur, altera per negationem, coefficientes ſubgraduales proficiſcuntur ex applicatione aggregati Poteſtatum ad differentiam eorum, in quos coefficiens eſt graduum.

Homogenea vero datæ menſuræ, ex applicatione aggregati factorum reciproce, a Poteſtate Radicis vnius in gradum alterius, ad prædictam cui altera applicatio facta eſt graduum in quos coefficiens eſt, differentiam. Id autem quoque obiter eſt demõſtrandũ.

Proponatur ſi quidẽ $\left.\begin{array}{l}\text{A poteſtas.}\\ + \text{B coefficiẽte in A gradũ}\end{array}\right\}$ ęquari z homogeneo.

Et rurſus eadẽ $\left.\begin{array}{l}\text{B coefficiẽ. in E gradũ.}\\ - \text{E poteſtate}\end{array}\right\}$ æquari eidẽ Z homogeneo.

Vnde intelligantur A & E pares gradus, atque adeo pares Poteſtates: oporteat autem æqualitatum harum conſtitutionem agnoſcere. Quoniam igitur quæ vni æquantur, æqualia ſunt inter ſe, manifeſtum fit ex iis quæ proponuntur.

$$\left.\begin{array}{l}\text{A poteſtatem}\\ + \text{E poteſtate.}\end{array}\right\}\text{æquari}\left\{\begin{array}{l}\text{B coefficienti in E gradum.}\\ - \text{B coefficiente in A gradum}\end{array}\right.$$

Itaque omnibus per E gradum, minus A gradu diuiſis.

$$\left.\frac{\text{E poteſtas} + \text{A poteſtate.}}{\text{E gradui.} - \text{A gradu.}}\right\}\text{æquabitur B coefficienti.}$$

Quod ipſum eſt quod primo loco enunciatur.

Porro quũ $\left.\begin{array}{l}\text{A poteſtas.}\\ + \text{B coefficiente in A gradum.}\end{array}\right\}$ æquetur z homogeneo:

In locũ B coefficiẽtis ſubgrad. ſubrogetur æquus valor, videlicet

$$\frac{\text{E poteſtas} + \text{A poteſtate}}{\text{E gradui} - \text{A gradu.}}$$

igitur A poteſtas plus $\left.\frac{\left.\begin{array}{l}\text{E poteſtate}\\ + \text{A poteſtate}\end{array}\right\}\text{in A gradum.}}{\text{E gradui.} - \text{A gradu.}}\right\}$ æqu. z homogeneo.

hoc eſt omnib. rite ordinatis. $\left.\frac{\text{A poteſtas in E grad.} + \text{E poteſtate in A grad}}{\text{E gradui.} - \text{A gradu.}}\right\}$ æq. z. homogen.

Quod ipſum eſt quod ſecundo loco enunciatur.

Et ancipites quidem dicimus omnino & in omni Poteſtatum ordine efficaces, quoniam agnita ſemel magiſtra, liberior eſt ad comitem tranſitus, vt ex zeteticis clarum eſt.

At Contradicentes & Inuerſæ, interdum efficaces ſunt, interdum minimè, idque è gradu Poteſtatis definitur, vt pote Contradicentes ſub latere, ita demum ſunt efficaces, ſi hærent in quadrato, vel Poteſtatibus exinde alternis, videlicet, quadrato-quadrato, cubo-cubo: & eo continuo ordine.

In baſe autem & Poteſtatibus exinde alternis, videlicet cubo, & quadrato-cubo, & eo continuo ordine, cenſemus pigras & inefficaces: quoniam nihil ſolatii præſidiiue afferunt Analyſtæ, quô ſibi pateat ad ἀντιβαλλόμενας facilior fæliciorue aditus.

Vt contra, Inuerſæ ſub latere, quum vtraque negata eſt, efficaces ſunt ſi hærent ſub cubo, quadrato-cubo, &c.

Cauſa eſt, quod quando differentia poteſtatum applicatur ad differentiam Radicum, quæ inde oritur magnitudo, æqua fit effectis continuè proportionalibus ab Radicibus iiſdem, ſerie vndique affirmata, in quocunque Poteſtatum ordine, ſecundum ea quæ expoſita ſunt in Capite de Geneſi poteſtatum à Binomia Radice.

At quum differentia Poteſtatum applicatur ad aggregatū Radicum, quæ inde oritur magnitudo, æqua fit effectis continuè proportionalibus proſaphæreticè alterne tantummodo, quum Poteſtates ſunt cubi, quadrato-cubi, & eo deinceps ordine in cedentes alterne.

Cæterum dum aggregatum Poteſtatum applicatur ad differentiam Radicum, nulla inde occurrit magnitudo æqua proportionalibus continuè effectis, ſiue per gradus ſcalæ numero pares, ſiue impares, Climacticæ magnitudines procedant.

Syncriticæ Doctrinæ Geometrica Phraſis.

CAP. XVII.

HÆc autem Syncritica iudicia, exornantur & expoliuntur per aliquot analogias, quibus excitatur Geometrica Mechanice, ac euidentior fit & paratior.

PRO-

PROPOSITIO. I.

SI fuerit ſeries trium proportionalium: eſt vt prima ad tertiam, ita quadratum é prima, ad quadratum è ſecunda.

Et ſi quatuor; Eſt vt prima ad quartam, ita cubus é prima, ad cubum è ſecunda.

Et ſi quinque: eſt vt prima ad quintam, ita quadrato-quadratum e prima, ad quadrato-quadratum e ſecundâ.

Et ſi ſex: eſt vt prima ſextá, ita quadrato-cubus é prima, ad quadrato cubum e ſecunda.

Et ſi ſeptem: eſt vt prima ad ſeptimá, ita cubo-cubus e prima, ad cubo-cubum e ſecunda.

Nam quadratum eſt poteſtas rationis duplicatæ, cubus, triplicatæ, Quadrato quadratum, quadruplicatæ, &c. vt alibi annotatum eſt: itaque Poteſtatibus ſingulis, ſuæ addicuntur proportionalium ſeries, ſecundum earum conditionem.

PROPOSITIO. II.

Si fuerit ſeries trium proportionalium: eſt vt prima ad tertiam, ita aggregatum quadratorum a ſingulis duabus primis, ad aggregatum quadratorum à ſingulis duabus poſtremis.

Et ſi quatuor: eſt vt prima ad quartam, ita aggregatũ cuborum a ſingulis tribus primis, ad aggregatum cuborum a ſingulis tribus poſtremis.

Et ſi quinque: eſt vt prima ad quintam, ita aggregatum quadrato-quadratorum a ſingulis quatuor primis, ad aggregatum quadrato-quadratorum a ſingulis duabus poſtremis.

Et ſi ſex: eſt vt prima ad ſextam, ita aggregatum qua-

drato-cuborum a ſingulis quinque primis, ad aggregatum quadrato-cuborum à ſingulis quinque poſtremis.

Et ſi ſeptem: eſt vt prima ad ſeptimam ita aggregatũ cubo cuborum à ſingulis ſex primis, ad aggregatum cubo-cuborum a ſingulis ſex poſtremis.

Quoniam enim vt prima ad tertiam, ita eſt quadratum primæ ad quadratum ſecundæ, vt vero quadratum primæ ad quadratum ſecundæ, ita quadratum ſecundæ ad quadratum tertiæ: Ergo per Synęreſin, eſt vt prima ad tertiam, ita quadratum primæ plus quadrato ſecundæ, ad quadratum ſecundæ plus quadrato tertiæ. Nec diſſimiliter in reliquis ſeriebus proportionalium & conditionarijs ſecundum rationem extremarum Poteſtatibus, licet arguere & rationari.

PROPOSITIO. III.

SI fuerit ſeries trium proportionalium: eſt vt prima ad tertiam, ita quadratum compoſitæ e duabus primis, ad quadratum compoſitæ é duabus poſtremis.

Et ſi quatuor: eſt vt prima ad quartam, ita cubus cõpoſitæ ex tribus primis, ad cubum compoſitæ ex tribus poſtremis.

Et ſi quinque: eſt vt prima ad quintam, ita quadrato-quadratum compoſitæ é quatuor primis, ad quadrato-quadratum compoſitæ e quatuor poſtremis.

Et ſi ſex: eſt vt prima ad ſextam, ita quadrato-cubus compoſitæ é quinque primis, ad quadrato-cubum compoſitæ é quinque poſtremis.

Et ſi ſeptem: eſt vt prima ad ſeptimam, ita cubo-cubus compoſitæ ex ſex primis, ad cubo-cubum compoſitæ ex ſex poſtremis.

Per Synereſin enim, eſt vt prima ad ſecundam, ita compoſita ex omnibus minus primâ, ad compoſitam ex omnibus minus alterâ

extremâ, quotcunque proportionalium ſit ſeries. In vnaquaque autem ſerie exponitur ſua poteſtas conditionaria: id eſt tantuplicatæ rationis, quam extremarum comparatio exigit.

PROPOSITIO. IIII.

SI fuerit ſeries trium proportionalium: eſt vt prima ad tertiam, ita differentia quadratorum a duabus primis, ad differentiã quadratorũ a duabus poſtremis.

Et ſi quatuor: eſt vt prima ad quartam, ita differentia cuborum à tribus primis alterne ſumptis, ad differentiam cuborum à tribus poſtremis alterne ſumptis.

Et ſi quinque: eſt vt prima ad quintam, ita differentia quadrato-quadratorum à quatuor primis alterne ſumptis, ad differentiam quadrato-quadratorum a quatuor poſtremis alterne ſumptis.

Et ſi ſex: eſt vt prima ad ſextam, ita differentia quadrato-cuborum a quinque primis alterne ſumptis, ad differentiam quadrato-cuborum a quinque poſtremis alterne ſumptis.

Et ſi ſeptem: eſt vt prima ad ſeptimam ita differentia cubo-cuborum a ſex primis alterne ſumptis, ad differentiam cubo-cuborum a ſex poſtremis alterne ſumptis.

PROPOSITIO. V.

SI fuerit ſeries trium proportionalium: eſt vt prima ad tertiam, ita quadratum differentiæ duarum primarũ, ad quadratum differentiæ duarum poſtremarum.

Et ſi quatuor: eſt vt prima ad quartam, ita cubus differentiæ trium primarum alterne ſumptarum, ad cubum differentiæ trium poſtremarum alterne ſumptarum.

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Et ſi quinque: eſt vt prima ad quintam, ita quadrato-quadratum differentiæ quatuor primarum alterné ſumptarum, ad quadrato-quadratum differentiæ quatuor poſtremarum alterné ſumptarum.

Et ſi ſex: eſt vt prima ad ſextam, ita quadrato-cubus differentiæ quinque primarum alterné ſumptarum, ad quadrato-cubum differentiæ quinque poſtremarum alterné ſumptarum.

Et ſi ſeptem: eſt vt prima ad ſeptimam, ita cubo-cubus differentiæ ſex primarum alterné ſumptarum, ad cubo-cubum differentiæ ſex poſtremarum alterne ſumptarum.

PROPOSITIO. VI.

SI fuerint quatuor continué proportionales, ſolidum compoſitum ex cubo quartæ, & triplo cubo ſecundæ, differt a ſolido compoſito ex cubo primæ & triplo cubo tertiæ, per cubum differentiæ extremarum.

Sint quatuor continuè proportionales B, D, F, G, & ſit G maior extrema, dico $\left.\begin{array}{l}\text{G cubum.}\\ +\text{D cubo ter}\\ -\text{B cubo}\\ -\text{F cubo ter.}\end{array}\right\}$ ęquari G — B cubo.

Enim-vero G — B cubus cōſtat $\left.\begin{array}{l}\text{G cubo}\\ -\text{B in G qu. ter}\\ +\text{B qu. in G ter}\\ -\text{B cubo.}\end{array}\right\}$ cōparetur autem $\left\{\begin{array}{l}\text{G cub.}\\ +\text{D c. ter}\\ -\text{B cub.}\\ -\text{F c. ter}\end{array}\right.$

Super eſt vt $\left.\begin{array}{l}\text{B quadratum in G ter}\\ -\text{B in G quadratum ter}\end{array}\right\}$ æquetur $\left\{\begin{array}{l}\text{D cubo ter.}\\ -\text{F cubo ter.}\end{array}\right.$

Id autem ita ſe habet, nam B quadratum in G eſt D cubus, & B in G quadratum, F cubus.

PROPOSITIO. VII.

SI fuerint quatuor continué proportionales, ſolidum compoſitum ex cubo quartæ & triplo cubo ſecundę, adiunctum ſolido compoſito ex cubo primæ & triplo cubo tertiæ, æquatur cubo aggregati extremarum.

Eadem enim viget quæ in antecedente Theoremate demonſtratio.

Æquationum Ancipitum conſtitutiua.

CAP. XVIII.

AGnitæ per Syncriſin ancipites æquationes, aut Plaſmatis reſolutione, aut denique zeteſeos vi, iſto modo fere ſe habent.

De affectis ſub depreſſiore gradu, ſeu Latere.

PROPOSITIO. I.

Si B in A
— A quadrato, } æquetur Z plano.

Eſt B compoſita e duobus lateribus, ſub quibus quod fit rectangulum, æquum eſt Z plano, & fit A latus maius minuſue.

Sunto duo latera 2. *&* 10. *dicetur* 12. N. — 1 Q *æquari* 20. *& fiet* 1 N. 2. *vel* 10.

PROPOSITIO. II.

Si B planũ in A
— A cubo } æquetur Z ſolido.

Eſt B planum compoſitum ex quadratis trium proportionaliũ; & Z ſolidum quod fit ductu alterius extremæ in aggregatum quadratorum à reliquis, & fit A prima vel tertia.

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Sunto proportionales. 2. *l.*20. 10. *dicetur* 124 N.—1 C. *æquari* 240. *& fit* 1 N. 2. *vel* 10.

PROPOSITIO. III.

Si B ſolidum in A —A quadrato-quadrato. } æquetur Z plano-plano.

Eſt B ſolidum compoſitum ex cubis quatuor continuè proportionalium: & z plano-planum quod fit ductu alterius extremæ, in aggregatum cuborum a tribus reliquis: & fit A prima vel quarta.

Sunto continuè proportionales. 2. *lc.*40. *lc.*200. 10.

Dicetur 1,248 N.—1 Q Q. *æquari* 2,480. *& fieri* 1 N. 2. *vel* 10.

PROPOSITIO. IV.

Si B plano-planum in A — A quadrato-cubo } æquetur z plano-ſolido.

Eſt B plano-planum compoſitum ex quadrato-quadratis quinque continuè proportionalium: & z plano ſolidum quod fit ductu alterius extremæ in aggregatum quadrato-quadratorum a quatuor reliquis: & fit A prima vel quinta.

Sunto continuè proportionales 2. *l*Q Q.80. *l.*20. *l* Q Q. 2,000. 10.

Dicetur 12,496 N.— 1 Q C. *æquari.* 24,960. *& fiet* 1 N. 2, *vel* 10.

PROPOSITIO. V.

Si B plano-ſolidum in A. A cubo-cubo. } æquetur z ſolido-ſolido.

Eſt B plano-ſolidum compoſitum ex quadrato-cubis ſex continue proportionalium: & z ſolido-ſolidum quod fit ductu alterius extremæ in aggregatum quadrato-cuborum a quinque reliquis: & fit A prima vel ſexta.

Sunto continuè proportionales:

2. *l.* Q C. 160. *l.* Q C. 800. *l.* Q C. 4,000. *l.* Q C. 20,000. 10.

Dicetur, 124,992. N.—1 C C. *æquari* 249,920. *& fit* 1 N. 2. *vel* 10.

De affectis ſub elatiore gradu, ſeu Lateri reciproco.

PROPOSITIO VI.

Si B in A quadratum,
—A cubo } æquetur Z ſolido.

Eſt B compoſita ex tribus proportionalibus, & Z ſolidū quod fit ductu alterius extremæ, in quadratum compoſitæ è duabus reliquis: & fit A compoſita è duabus primis: vel è duabus poſtremis.

Sint proportionales, 1. 2. 4.

Dicetur 7 Q. —1 C. *æquari* 36. *& fit* 1 N. 3. *vel* 6.

PROPOSITIO. VII.

Si B in A cubum
— A quadrato-quadrato. } æquetur Z plano-plano.

Eſt B compoſita ex quatuor continuè proportionalibus, & Z plano-planum, quod fit ductu alterius extremæ in cubum compoſitæ e tribus reliquis: & fit A, compoſita e tribus primis, vel è tribus poſtremis.

Sint continue proportionales 1. 2. 4. 8.

Dicetur 15 C. —1 QQ. *æquari* 2,744. *& fiet* 1 N. 7. *vel* 14.

PROPOSITIO. VIII.

Si B in A quadrato-quadratum.
—A quadrato-cubo. } æquetur Z plano-ſolido.

Eſt B compoſita ex quinque continuè proportionalibus: & Z plano-ſolidū quod fit ductu alterius extremæ in quadrato-quadratum compoſitæ e quatuor reliquis: Et fit A compoſita ex quatuor primis vel quatuor poſtremis.

Sint continue proportionales 1. 2. 4. 8. 16,

31 QQ. — 1 QC *æquabitur* 810, 000. *& fit* 1 N. 15. *vel* 30.

PROPOSITIO. IX.

Si B in A quadrato-cubum
— A cubo-cubo. } æquetur Z ſolido-ſolido.

Eſt B compoſita ex ſex continue proportionalibus, & Z ſolido-ſolidum quod fit ductu alterius extremæ in quadrato-cubum cõpoſitæ a reliquis: & fit A compoſita ex quinque primis vel e quinque poſtremis.

Sint continuè proportionales 1. 2. 4. 8. 16. 32.
6 3 Q C. — 1 C C. *æquatur.* 9 16, 1 32, 8 32. *& fit* 1 N. 31. *vel* 62.

De affectis ſub gradibus intermedijs quæ per interpretationem deprimuntur.

PROPOSITIO. X.

Si B planum in A quadratum.
— A quadrato-quadrato. } æquetur Z plano-plano.

Eſt B planum compoſitum ex duobus quadratis duorum laterum, quorum primi ductu in ſecundum fit Z plano-planum: & fit A quadratum maius duorum, minuſue.

Sint latera 1. 4. *dicetur* 17 Q. — 1 Q Q *æquari* 16. *& fit* 1 N. 1. *vel* 4.
Quod ſi vnus numerus intelligatur quadratum, Radixne plana.
17 N. — 1 Q. *æquabitur* 16. *& fit* 1 N. 1. *vel* 16.

PROPOSITIO. XI.

Si B ſolidum in A cubum.
— A cubo-cubo. } æquetur Z ſolido-ſolido.

Eſt B ſolidum compoſitum ex duobus cubis, quorum primi ductu in ſecundum fit Z ſolido-ſolidum, & fit A cubus maior duorum minorue.

Sunto latera 1. 8. *dicetur* 513 C. — 1 C C. *æquari* 512. *& fit* 1 N. 1. *vel* 8.
Quod ſi 1 N. *intelligatur cubus Radixne ſolida,* 513 N. — 1 Q. *æquabitur* 512. *& fit* 1 N. 1 *vel* 512.

PROPOSITIO. XII.

Si B plano-planum in A quadratum, — A cubo-cubo. æquetur Z ſolido-ſolid. Eſt B planum compoſitum ex quadratis trium planorum proportionalium, & z ſolido-ſolidum quod fit ab vno plano in aggregatum quadratorum à duobus reliquis planis: & fit A quadratum, maius minuſue.

Sunto tria plana proportionalia. 1. 2. 4.

Dicetur. 21 *Q.* — 1 *C C. æquari* 20. *& fit* 1 *N.* 1. *vel* 2.

Quod ſi 1 *N. intelligatur quadratum Radixue plana,* 21. *N.* — 1 *C. æquabitur* 20. *& fiet* 1 *N.* 1. *vel* 4.

PROPOSITIO. XIII.

Si B planum in A quadr.quadratum, — A cubo-cubo. æquetur z ſolido-ſolido. Eſt B planum compoſitum a tribus planis proportionalibus, & z ſolido-ſolidum quod fit ab vno plano, in quadratū compoſiti ex duobus reliquis: & fit A quadratum, maius duorum minuſue.

Sint tria proportionalia plana 5. 20. 80.

Dicetur 105 *Q Q.* — 1 *c c. æquari* 50,000. *et fit* 1 *N.* 5. *vel* 10.

Inuentio autem trium planorum proportionalium, quorum medium adſcito ſiue primo ſiue poſtremo, ſit quadratum numero, patet ex hac ſerie.

$$\frac{\text{B quadrato-quadratum.}}{\text{B quadrato} + \text{G quadrato}} \qquad \frac{\text{B quad. in G. quad.}}{\text{B quad.} + \text{G quad.}} \qquad \frac{\text{G quad. quad.}}{\text{B quad.} + \text{G quad.}}$$

Dictante videlicet zeteſi.

Sit enim medium planorum E planum: & primum ſtatuatur B quadratum minus E plano, poſtremum, G quadratum minus E plano: quum igitur primo plano addetur E planum, fiet quadratū, nempe B quadratum: reſtat igitur vt ea tria plana proportionalia ſint, & conſequenter quod a medio fit in ſe, æquetur ei quod fit ab extremis, quâ comparatione ſecundum artem initâ.

$$\frac{\text{B quad. in G quad.}}{\text{B quad.} + \text{G quad.}}$$ Inuenitur æquari E plano.

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Sit B 1. G 2. plana quæſita erunt $\frac{1}{5}$ $\frac{4}{5}$ $\frac{16}{5}$

Medium adſcito primo facit 1. adſcito vero ſecundo 4. eadem plana ducantur in aliquod quadratum vt pote 25. fiunt 5. 20. 80. qualia ad exemplum adſumpta ſunt.

De reliquis.

PROPOSITIO. XIIII.

Si B ſolidum in A quad.
—A quadrato-cubo } æquetur z plano-ſolido.

Eſt B ſolidum conſtans cubo compoſitæ è duabus primis in ſerie trium proportionalium, plus cubo compoſitæ è duabus poſtremis, & inſuper ſolido, quod fit ductu alterius extremarum in quadratum compoſitæ ex ſecunda & tertia.

Et z plano ſolidũ quod fit â B ſolido minus cubo à duabus primis, in quad. cõpoſitæ ex prima & ſecunda, vel a cubo compoſitæ ex ſecunda & tertia, plus ſolido ſub tertia, & quadrato compoſitæ ex prima & ſecũda, in quadratum compoſitæ ex prima & ſecunda:

Et fit A compoſita ex duabus primis, vel compoſita ex duabus poſtremis.

Sint proportionales 1. 2. 4. 279Q.—1QC *æquabitur* 2,268. *& fit* 1N. 3. *vel* 6.

PROPOSITIO. XV.

Si B planum in A cubum.
—A quadrato-cubo. } æquetur z plano-ſolido.

Eſt B planum conſtans quadrato compoſitæ à tribus primis in ſerie quatuor cõtinuè proportionalium, plus quadrato compoſitæ è tribus poſtremis, minus plano quod fit â tertia, in compoſitam ex tertia, ſecunda, & prima: vel à ſecunda in compoſitam ex ſecunda, tertia & quartâ.

Et Z plano-ſolidum quod fit à B plano, minus quadrato compoſitæ ex tribus primis, in cubum compoſitæ è tribus poſtremis: vel abs B plano, minus quadrato compoſitæ ex tribus poſtremis, in cubum compoſitæ ex tribus primis, & fit A compoſita ex tribus primis, vel compoſita ex tribus poſtremis.

Sint proportionales continuè 1. 2. 4. 8.
217 C—1 QC. *æquabitur* 57,624. *& fit* 1 N. 7. *vel* 14.

Æqualitatum Contradicentium Conſtitutiua.

CAP. XIX.

Contradicentium autem conſtitutio eſt huiuſmodi.

PROPOSITIO. I.

Si A quadratum
+ B in A. } æquetur Z plano.

Et rurſus } E quadratum
— B in E } æquetur z plano.

Sunt duo latera quorum differentia eſt B quod autem ſub eis fit, æquum eſt z plano, & fit A minus latus & maius.

Sunto latera 1. 2. 1 Q. + 1 N. *æquatur* 2. *&* *fit* 1 N. 1.

Rurſus 1 Q — 1 N. *æquabitur* 2. *&* *fit* 1 N. 2.

PROPOSITIO. II.

Si A quadrato-quadratum
+ B ſolido in A. } æquetur z plano-plano.

Et rurſus } E quadrato-quadr.
— B ſolido in E } æquetur z plano-plano.

Sunt quatuor continuè proportionales, a quibus cubi alternè ſumpti differunt per B ſolidum. Fit autem z plano-planum, ductu alterius extremæ in differentiam cuborum a reliquis alterne ſumptorum: & eſt A prima minor inter extremas, E quarta.

Sunto continuè proportionales 1. *lc.* 2. *lc.* 4. 2.

1 Q Q + 5 N. *æquabitur* 6. *&* *fit* 1 N. 1.

Rurſus 1 Q Q — 5 N. *æquabitur* 6 *&* *fit* 1 N. 2.

PROPOSITIO. III.

Si A cubo-cubus.
+ B plano ſolido in A. } æquetur z ſolido-ſolido.

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E cubo-cubus.
& rurſus } — B plano-ſolido in E { æquetur z ſolido-ſolido.

Sunt ſex continuè proportionales, a quibus quadrato-cubi alterne ſumpti, differunt per B plano-ſolidum. Fit autem z ſolido-ſolidum, ductu alterius extremæ in differentiam quadrato-cuborũ a reliquis alterne ſumptorum, & eſt A prima minor inter extremas, E ſexta.

Sunto proportionales continuè 1. *lQC.*2. *lQC.*4. *lQC.*8. *lQC.*16. 2.
1 *C C.* + 21 *N. æquatur* 22. *& fit* 1 *N.* 1.
Et rurſus 1 *C C.* — 21 *N. æquabitur* 22. *& fit* 1 *N.* 2.

PROPOSITIO IIII.

Si A quadrato-quadratum.
\+ B in A cubum. { æquetur z plano-plano.

E quadrato-quadratum
Et rurſus } — B in E cubum. { æquetur z plano-plano.

Sunt quatuor continue proportionales, quarum alterne ſumptarum differentia eſt B: & fit z plano-planum, ductu vtriuſuis extremæ, in cubum differentiæ reliquarum alterne ſumptarum, & dum intelligitur prima, minor inter extremas, eſt A differentia alterne ſumpta trium primarum, E differentia trium poſtremarum.

Sunto proportionales continuè 1. 2. 4. 8.
1 *QQ.* + 5 *C. æquabitur* 216. *& fit* 1 *N.* 3.
Et rurſus 1 *QQ.* — 5 *C. æquabitur* 216. *& fit* 1 *N.* 6.

PROPOSITIO. V.

Si A cubo-cubus
\+ B in A quadrato-cubum { æquetur z ſolido-ſolido.

E cubo-cubus.
Et rurſus } — B in E quadrato-cubum { æquetur z ſolido-ſolido.

Sunt ſex continue proportionales, quarum alterne ſumptarum differentia eſt B: fit autem z ſolido-ſolidum, ductu vtriuſuis extremæ in quadrato-cubum differentię reliquarum alterne ſumptarũ, & dum intelligitur prima, minor inter extremas, fit A differentia alterne ſũpta quinq; primarũ, & E differẽtia quinq; poſtremarum.

Sunto continuè proportionales. 1. 2. 4. 8. 16. 32.
1 *CC.* + 21 *QC. æquatur* 5, 153, 632. *& fit* 1 *N.* 11.
Et rurſus 1 *cc.* — 21 *QC. æquabitur* 5, 153, 632. *& fit* 1 *N.* 22.

Æqualitatum Inuerſarum Conſtitutiua.

CAP. XX.

Inuerſarum denique Syſtatica ſunt hæc.

PROPOSITIO. I.

Si B planũ in A
— A cubo } æquetur Z ſolido.

E cubus
& rurſus } — B plano in E } æquetur Z ſolido.

Sunt tres proportionales, a quibus quadrata alternè ſumpta, differunt per B planum: fit autem z ſolidum, ductu alterius extremæ in differentiam quadratorum a reliquis: & eſt A prima minor inter extremas, E tertia.

Sunto proportionales 1. *l.*2. 2.
3 *N.* — 1 *C æquabitur* 2. *& fit* 1 *N.* 1.
Et rurſus 1 *C* — 3 *N. æquabitur* 2. *& fit* 1 *N.* 2.

PROPOSITIO. II.

Si B plano-planũ in A
— A quadrato-cubo. } æquetur z plano-ſolido.

E quadrato-cubus
Et rurſus } — B plano-plano in A } æquetur z plano-ſolido.

Sunt quinque continue proportionales longitudines, a quibus quadrato-quadrata alterne ſumpta, differunt per B plano planum. Fit autem z plano-ſolidum, ductu alterius extremæ in differentiam quadrato-quadratorum a reliquis alterne ſumptis: & fit A prima minor inter extremas, E quinta.

Sint continue proportionales 1, *l.QQ.*2. *l.QQ.*4. *l.QQ.*8. 2.
11 *N.* — 1 *QC. æquatur* 10. *et fit* 1 *N.* 1.
Et rurſus 1 *QC.* — 11 *N. æquatur* 10. *et fit* 1 *N.* 2.

PROPOSITIO. III.

Si B in A quadratum
+A cubo. } æquetur z ſolido.

& rurſus } B in E quadratum
—E cubo. } æquetur z ſolido.

Sunt tres proportionales, quarum alternè ſumptarum differentia eſt B: fit autem z ſolidum ductu alterutrius extremæ in quadratum differentiæ reliquarum alterne ſumptarum: & dum intelligitur prima, minor inter extremas, eſt A differentia alterné ſumpta duarum primarum, E differentia duarum poſtremarum.

Sunto proportionales, 1. 2. 4.
3 *Q* + 1 *C æquatur* 4. *& fit* 1 N. 1.
Et rurſus 3 Q — 1 C. *æquatur* 4. *& fit* 1 N. 2.

PROPOSITIO. IV.

Si B in A quadrato-quadratum
+A quadrato-cubo. } æquetur z plano-ſolido.

& rurſus } B in E quadrato-quadratum.
— E quadrato-cubo. } æquetur z pl. ſolido.

Sunt quinque proportionales, quarum alternè ſumptarum differentia eſt B: fit autem Z plano-ſolidum ductu alterutrius extremæ in quadrato-quadratum differentiæ reliquarum alterne ſumptarum: & dum intelligitur prima, minor inter extremas, eſt A differentia alterne ſumpta quatuor primarum, E differentia quatuor poſtremarum.

Sunto proportionales 1. 2. 4. 8. 16.
11 *Q Q.* + 1 Q C. *æquatur* 10, 000. *& fit* 1 N. 5.
Et rurſus 11 *Q Q* — 1 *Q C. æquatur* 10, 000. *& fit* 1 N. 10.

PROPOSITIO. V.

Si B planum in A cubum.
+A quadrato-cubo. } æquetur z plano-ſolido.

& rurſus } B planum in E cub.
— E quadrato-cubo. } æquetur Z plano-ſolido.

Sunt ſex proportionales continue, quarum extremæ ductæ in differentiam quartæ & primæ, faciunt B planum: fit autẽ Z pl. ſolid. ex cubo differentiæ quartæ & primæ, in B pl. plus eiuſdem differentiæ quadrato: vel ex cubo differentiæ quintæ & ſecundę in B planũ, multatum illius differentiæ inter quintã & ſecundã quad. & quum prima intelligitur minor inter extremas, fit A differentia quartæ & primæ, & E differentia quintæ & ſecundæ.

Sunto proportionales 1. 2. 4. 8. 16. 32.
231 C + 1 QC *æquatur* 96,040, *& fit* 1 N. 7.
Et rurſus 231 C — 1 QC. *æquatur* 96,040. *& fit* 1 N. 14.

PROPOSITIO. VI.

Si B ſolidum in A quad. + A quadrato-cubo. } æquetur Z plano ſolido.
& rurſus B ſolidum in E quadratum — E quadrato-cubo. } æquetur Z plano-ſolido.

Sunt ſex proportionales continue, quarum extremæ ductæ in quadratum a differentiâ tertiæ & primæ, faciunt B ſolidum: fit autem Z plano-planum à B ſolido, plus cubo à differentia tertiæ & primæ, in quadratum differentiæ eiuſdem, vel a B ſolido minus cubo a differentia ſecundæ, & quartæ, in quadratum differentiæ illius inter ſecundam et quartam, et quum prima intelligitur minor inter extremas, fit A tertia minus primâ, E quarta minus ſecunda.

Sunto proportionales 1. 2. 4, 8. 16. 32.
297 Q + 1 QC. *æquatur* 2,916 *& fit* 1 N. 3.
Et rurſus 297 Q — 1 QC. *æquatur* 2,916 *& fit* 1 N. 6.

Alia rurſus æqualitatum Inuerſarum Conſtitutiua.

CAP. XXI.

PROPOSITIO. I.

Si B planum in A — A cubo } æqueturZ ſolido.
& rurſus E cubus — B plano in E. } æqueturZ ſolido.

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Sunt tres proportionales, quarum quadrata iuncta, conficiunt B planum, ſumma autem extremarum ducta in mediæ quadratũ, vel altera extremarum in aggregatum quadratorum a duabus reliquis, facit z ſolidum: & fit A alterutra extremarum, E vero earundem ſumma.

Sunto proportionales 1. 2. 4.

21 N. — 1 *C. æquatur* 20. *& fit* 1 N. 1. *vel* 4.

Et rurſus 1 C — 21 N. *æquatur* 20. *& fit* 1 N. 5.

PROPOSITIO. II.

Si B in A quadratum. — A cubo. } æquetur z ſolido.

& rurſus } B in E quadratum + E cubo } æquetur z ſolido.

Sunt tres proportionales Radices, quarum ſumma eſt B, compoſita autem e duabus primis adiecta compoſitæ a duabus reliquis, dum ducitur in mediæ quadratum, uel altera extremarum in quadratum compoſitæ a duabus reliquis, facit z ſolidum. & fit A alterutra prædictarum compoſitarum, E media inter extremas.

Sunto proportionales 1. 2. 4.

7 Q. — 1 C. *æquatur* 36. *et fit* 1 N. 3. *vel* 6.

Et rurſus 7 Q. + 1 C. *æquatur* 36. *et fit* 1 N. 2.

FRANCISCI
VIETAE FONTENAEENSIS
DE ÆQVATIONVM EMENDATIONE
TRACTATVS SECVNDVS.

De Solennibus quinque modis præparandarum Æquationum, aduerſus earum in numeris ΔΥΣΜΗΧΑΝΙΑΝ.

Ac Primum.

De Expurgatione per Vncias, quæ Remedium eſt aduerſus Πολυπαθείαν.

CAPVT. I.

AGNITA æquationum conſtitutione, Analyſta ad præparandum eas quæ ſuam alioquin mechanicē reſpuant, aut demum ægré ſubeunt, tuto ſe confert, & ſua præparatione efficit εὐμήχανας. & præparandi quidem generalis doctrina eſt, vt noua zeteſis inſtituatur, vel Plaſmatis, aut Syncriſeos veſtigia repetantur, ac denique nullus non tentetur tranſmutandi modus: ſed non deſunt Analyſtæ ſingula-

ria & Topica remedia, aduerſus vicia quæque æquationum, impedimentaue, quo minus fœliciter ſiue re, ſiue numero explicentur: fere autem quæ proſunt Geometræ ad εὐμηχανίαν, proſunt & Arithmetico, vel etiam è contrà. At etiam de effectionibus Geometricis dicetur ſpecialius ſuo loco: nunc autem circa numeroſam Analyſin magis eſſe intentum, noſtri eſt inſtituti.

Præparationum igitur præſertim in numeris, ſolennes & ſpeciales modi ſunt fere quinque.

I. Expurgatio per Vncias.

II. Tranſmutatio Πρώτον ἔσχατον.

III. Anaſtrophe.

IIII. Iſomeria.

V. Climactica Parapleroſis.

Omnino aduerſus πολυπαθείαν tutiſſimum ac paratiſſimum remedium eſt, Expurgatio per Vncias.

Eſt autem ſpecies tranſmutationis per additionem, vel ſubductionē. Hâc æquationes Poteſtatum adfectarum ſub extremo paradico gradu, quem ſuſtinet coefficiens Radici homogenea, regulariter eâ adfectione liberantur, ſaluâ numerorum Symmetria. & in quadratis eſt Diahemiſy, in cubis Diatritemorion, in quadrato-quadratis Diatetartemorion, in quadrato-cubis Diapentemoriō, in cubo cubis, Diectemorion, & eo in infinitum ordine: quoniam adfectiones ſub eo gradu, proficiſcuntur à cremento, decremento ue, quo intelligitur adfecta Radix de quâ, cuiuſue Poteſtate, purâ vel adfectâ, primum proponebatur æquatio.

Huius autem crementi decrementiue duplum, eſt coefficiens ſub latere in quadratis: & triplum, coefficiens ſub quadrato in cubis: & quadruplum, coefficiens ſub cubo in quadrato-quadratis: & quintuplum coefficiens ſub quadrato-quadrato, in quadrato-cubis: & ſextuplum denique, coefficiens ſub quadrato-cubo, in cubo-cubis: & eo in infinitum ordine.

Itaque ad delendum Plaſma, contraria retrogradaque viâ ſumuntur Vnciæ conditionariæ coefficientium Radici homogenearum: in quadratis videlicet, ſemis, in cubis triens, in quadrato-quadratis quadrans, in quadrato-cubis quintans, in cubo-cubis ſextans, & eo continuo ordine: atque illis vncijs conditionarijs

adficitur Radix, atque adeo arte fit tranſmutatio.

Totius itaque operis ſtructura, tres caſuum differentias admittit.

Primo, ſit A Poteſtas adfecta per adiunctionem homogenei ſub B coefficiente, & A Parodico extremo gradu: quoniam igitur ea adfectio affirmata eſt, adficietur Radix affirmate a conditionarijs B coefficientis vncijs, & ita adfecta, ſtatuetur E, vnde E minus conditionarijs B coefficientis vncijs, æquabitur A, ſub qua noua ſpecie, æquatio primum propoſita dirigetur, & ordinabitur noua, quæ omnino eueniet immunis ab adfectione ſub gradu extremo: vt opus comprobabit.

Secundo: ſit A Poteſtas adfecta per multam homogenei ſub B coefficiente, & A extremo Parodico gradu: quoniam igitur ea adfectio negata eſt, adficietur A negate a conditionarijs B coefficiētis vncijs, & ita adfecta ſtatuetur eſſe E, vnde E plus conditionariis illis B coefficientis vncijs æquabitur A, ſub quâ nouâ ſpecie, æquatio primam propoſita dirigetur & ordinabitur noua, quæ omnino eueniet immunis ab adfectione ſub gradu extremo: vt opus comprobabit.

Poſtremò: ſit homogeneum ſub B coefficiente, & A extremo gradu, adfectum per multam A Poteſtatis, quoniam igitur Poteſtas potius adficit quam adficitur, vt pote quæ negatur de homogeneo adfectionis, conditionariæ B coefficientis vnciæ multabuntur A Poteſtate, & ſtatuentur eſſe E, vnde eædem B coefficientis vnciæ minus Radice E, æquabuntur A, ſub quâ noua ſpecie ęquatio primum propoſita dirigetur, & ordinabitur noua, quæ omnino eueniet immunis ab adfectione ſub gradu extremo: vt opus comprobabit.

Exemplum in Quadrato.

PRoponatur A quadratum plus B in A, æquari z plano: quoniam igitur in Climacticarum magnitudinum ordine, latus quadrato proxime ſuccedit, proponitur autem hic quadratum adfectum ſub latere, omnino æqualitas Plaſmatica eſt, aliundeue effcta: eſt autem affectio illa adfirmata, itaque Plaſma fuit per additionem ſemiſſis coefficientis ſub latere, prout conditio quadrati, quæ Poteſtas eſt rationis duplicitatæ expoſcit.

Ad tol'endum igitur Plaſma, fiat Expurgatio Diahemiſy: & idcirco A + B eſto E, ergo E — B erit A, & conſequenter quadratum abs E — B adiunctū plano ſub B bis in E — B, æquabitur z plano, per ea quæ proponuntur: æqualitas igitur de E, ſecundum artem ordinetur: omnibus rite peractis, deprehendetur E quadr. æquari z plano, plus B quadrato: quę quidem noua æquatio, pura eſt ab adfectione ſub latere, quâ primum propoſita æquatio obruebatur: quum autem innoteſcet E, non poterit A ignorari, propter datam inter eas Radices differentiam: præſtat ſi quidem E ipſi A per longitudinem B. quare factum eſt quod oportuit.

Exemplum in cubo.

PRoponatur A cubus plus B in A quadratum ter, plus D plano in A, æquari z ſolido: quoniam igitur in magnitudinum Climacticarum ordine, quadratum cubo proximé ſuccedit, proponitur autem hic cubus adiectus vtique ſub quadrato, omnino æqualitas Plaſmatica eſt, aliunde ue effecta. eſt autem adfectio illa adfirmata, quare Plaſma fuit per additionem trientis B coefficientis, prout conditio cubi, quæ Poteſtas eſt rationis triplicatæ, expoſcit. ad tollendum igitur Plaſma, fiat Expurgatio Diatritemorion, & idcirco A + B eſto E, ergo E — B valebit A: & conſequenter, effectus abs E — B cubus, adiunctus ſolido abs B ter in effingendum quoque abs E — B quadratum, ac denique adiunctus ſolido abs D plano in E — B, æquabitur Z ſolido per ea quę proponuntur. æqualitas igitur de E ſecundum artem ordinetur, omnibus rite peractis deprehendetur.

E cubus
+ D plano } in E } æquari { Z ſolido. + D plano in B
— B quadrato ter. — B cubo bis.

Quæ quidem noua æquatio pura eſt ab adfectione ſub quadrato, quà primum propoſita obruebatur: quum autem innoteſcet E, non poterit A ignorari, propter datam inter eas radices differentiam: præſtat ſi quidem E ipſi A, per longitudinem B. quare factum eſt quod oportuit.

Quod ſi proponitur æqualitas Poteſtatis adfectæ per negationem directè, vt quum A quadratum minus B in A bis, æquatur z

plano: vel A cubus minus B in A quadratū ter, adæquatur z ſolido, A — B ſtatuetur eſſe E, & æqualitas de E, ex poſitis veſtigiis ordinabitur.

Si denique proponitur æqualitas Poteſtatis adfectæ per negationem inuerſe, vt quum B in A bis minus A quadrato, adæquatur z plano: vel B in A quadratum ter, minus A cubo adæquatur z ſolido B — A ſtatuetur eſſe E, & æqualitas de E, ſimiliter ex poſitis veſtigiis ordinabitur.

Sic quadrata omnia adfecta, reducuntur ad pura: adfecti qualitercunque cubi, ad cubos adfectos tantum ſub latere: adfecti qualitercunque quadrato-quadrata, ad quadrato-quadrata adfecta duntaxat ſub latere & quadrato: adfecti qualitercunque quadrato-cubi, ad quadrato-cubos adfectos duntaxat ſub latere, quadrato & cubo: & eo deinceps ordine.

Credebant autem Antiqui, quoniam hac reductione, æquationes quadraticas omnino expurgabant, & fœliciter explicabant, in vlterioribus quoque Climacticis accidere vt omnino expurgarētur, & â Canonicâ purarum reſolutione negocium omne Mechanicum deriuare tentarunt, adeo obſtinate, vt aliunde methodum explicandi æquationes adfectas, non exquiſierint.

Itaq; excruciarunt ſe fruſtra & bonas horas Mathematices quam colebant diſpendio, abſumpſerunt. In ſumma Methodus illa explicandi æquationes adfectas quadraticas, non eſt Catholica: Catholica quidem eſt methodus expurgandarum ęquationum adfectione ſingulari, ſalua numerorum Symmetria, ſed non adfectione omni, vt deinceps veterum pertinaciæ non ſit inhęrendum.

Iuuat autem de ſingulis iſtiuſmodi reductionum formulis, ſingula concepiſſe Theoremata, & ea in artem & vſum proferre, qualia ſunt quæ ſequuntur.

De Reductione Quadratorum adfectorum ad pura. Formulæ tres.

I.

Si A quadratum + B bis in A ęquetur Z plano.

A + B eſto E. igitur E quadratum, æquabitur Z plano + B quadr.

Conſectarium.

Itaque, l. $\left\{\begin{matrix} \text{Z plani.} \\ + \text{B quadrato.} \end{matrix}\right\}$ — B, fit A, de qua primum quærebatur.

Sit B 1. Z planum 20. A 1 N.

1 Q + 2 N. *æqu. tur* 20. *& fit* 1 N. *l.* 21. — 1.

I I.

Si $\begin{matrix} \text{A quadratum} \\ - \text{B in A bis.} \end{matrix}\Big\}$ æquetur Z plano.

A — B eſto E. igitur E quadratum, æquabitur $\left\{\begin{matrix} \text{Z plano} \\ + \text{B quadrato.} \end{matrix}\right.$

Conſectarium.

Itaque, l. $\left\{\begin{matrix} \text{Z plani.} \\ + \text{B quadrato.} \end{matrix}\right\}$ + B, fit A, de qua primum quærebatur.

Sit B 1. Z planum 20. A 1 N.

1 Q — 2 N. *æquabitur* 20. *& fit* 1 N *l.* 21 + 1.

I I I.

Si $\begin{matrix} \text{D bis in A} \\ - \text{A quadrato} \end{matrix}\Big\}$ æquetur Z plano.

D — E, vel D + E eſto A. E quad. æquabitur. $\left\{\begin{matrix} \text{D quad.} \\ - \text{Z plano.} \end{matrix}\right.$

Conſectarium.

Itaque, D. minus, pluſue l. $\left\{\begin{matrix} \text{D quadrati} \\ - \text{Z plano.} \end{matrix}\right\}$ fit A. de qua primũ quærebatur.

Sit D 5. Z planum 20. A 1 N

10 N. — 1 Q. *æquatur* 20. *& fit* 1 N. 5 — *l* 5, *vel* 5 + *l* 5.

De Reductione Cuborum ſimpliciter adfectorum ſub Quadrato, ad Cubos ſimpliciter adfectos ſub Latere. Formulæ tres.

I.

Si $\begin{matrix} \text{A cubus.} \\ + \text{B ter in A quadratum.} \end{matrix}\Big\}$ æquetur Z ſolido.

A + B eſto E. $\left\{\begin{matrix} \text{E cubus} \\ - \text{B quad ter in E} \end{matrix}\right\}$ æquabitur $\left\{\begin{matrix} \text{Z ſolido} \\ - \text{B cubo bis.} \end{matrix}\right.$

1 C. + 6 Q. *æquatur* 1,600. *eſt* 1 N. 10.

1 C — 12 N. *æquatur* 1,584 *eſt* 1 N. 12.

Ad Arithmetica non incongrué σημεῖον aliquod ſuperimponitur

notis alteratæ Radicis, ad differentiam notarum eius, de qua primum quærebatur.

II.

Si A cubus
— B ter in A quadrat. } æquetur z ſolido.

A — B eſto E. E cubus.
— B quadrato ter in E } æquabitur { Z ſolido.
+ B cubo bis.

1 C — 6 Q. æquetur 400. eſt 1 N. 10.
1 C — 12 N. æquatur 416. eſt 1 N. 8.

III.

Si B ter in A quadratum.
— A cubo } æquetur Z ſolido.

A — B eſto E. B quadratum ter in E.
— E cubo } æquabitur { Z ſolido.
— B cubo bis.

vel B — A eſto E. B quadratum ter in E.
— E cubo } æquabitur { B cubo bis.
— Z ſolido.

21 Q. — 1 C æquatur 972. & eſt 1 N. 9. vel 18.
147 N — 1 C æquatur 286. & eſt 1 N 2 vel 11.

9 Q. — 1 C æquatur 28. & eſt 1 N. 2.
27 N. — 1 C æquatur 26. & eſt 1 N. 1.

De Reductione Cuborum adfectorum tam ſub Quadrato quam latere, ad Cubos adfectos ſimpliciter ſub Latere. Formulæ ſeptem.

I.

Si A cubus
+ B ter in A quadratum
+ D plano in A } æquetur z ſolido.

A + B eſto E. E cubus.
± D plano.
— B quadrato ter. } in E } æquabitur { Z ſolido.
± D plano in B.
— B cubo bis.

1 C. + 30 Q. + 330 N. æquatur 788. & eſt 1 N. 2.
1 C. + 30 N æquatur 2,088. & eſt 1 N. 12.

1 C + 24 Q. + 132 N. æquatur 368. & eſt 1 N. 2.
1 C — 60 N. æquatur 400. & eſt 1 N. 10.

1 C. + 30 Q. + 4 N. æquatur 1,320. & eſt 1 N. 6.
296 N. — 1 C æquatur 640. & eſt 1 N. 16.

II.

Si A cubus
+ B ter in A quadratum.
— D plano in A. } æquetur Z ſolido.

A + B eſto E. { E cubus
— B quadrato 3.
— D plano } in E } æquabitur { Z ſolido.
— B cubo-bis
— D plano in B.

1 *C.* + 6 *Q.* — 48 *N* *æquatur* 512. *& eſt* 1 N. 8.
1 *C* — 60 *N.* *æquatur* 400. *& eſt* 1 N. 10.

1 *C* + 30 *Q.* — 48 *N.* *æquatur* 32. *& eſt* 1 N. 2.
348 *N* — 1 *C.* *æquatur* 2,448 *& eſt* 1 N. 12.

III.

Si A cubus
— B ter in A quadratum
+ D plano in A } æquetur Z plano.

A — B eſto E { E cubus.
— B quadrato ter
+ D plano } in E } ęquabitur { Z ſolido
+ B cubo bis.
— D plano in B.

1 *C* — 30 *Q.* + 330 *N.* *æquatur* 1,368. *& eſt* 1 *N.* 12.
1 *C* + 30 *N.* *æquabitur* 68. *& eſt* 1 *N.* 2.

1 *C* — 12 *Q* + 28 *N.* *æquatur* 80. *& eſt* 1 *N.* 10.
1 *C* — 20 *N.* *æquatur* 96. *& eſt* 1 *N.* 6.

1 C. — 18 Q + 88 N. *æquatur* 80. *& eſt* 1 N. 10.
20 N. — 1 C. *æquatur* 16. *& eſt* 1 N. 4.

vel B — A eſto E. { B quadratum ter.
— D plano.
— E cubo. } in E } æquabitur { Z ſolido
+ B cubo bis
— D plan. in B.

1 C. — 30 Q. + 200 *N.* *æquatur* 336. *et eſt* 1 *N.* 6.
100 *N.* — 1 C *æquatur* 336. *et eſt* 1 *N.* 4.

1 C — 30 Q + 280 *N.* *æquatur* 704. *et eſt* 1 *N.* 4.
1 C — 20 *N.* *æquatur* 96. *et eſt* 1 *N.* 6

1 C — 30 Q + 330 *N.* *æquatur* 1,232. *et eſt* 1 *N.* 8.
1 C + 30 *N.* *æquatur* 68. *et eſt* 1 *N.* 2.

IIII.

IV.

Si A cubus.
— B ter in A quadratum
— D plano in A.
} æquetur Z ſolido.

A — B eſto E, { E cubus
— B quadrato ter.
— D plano. } in E } æquabitur { Z ſolido.
+ B cubo bis
+ D plano in B.

1 C — 6 Q. — 28 N *æquetur* 120. *& eſt* 1 N. 10.
1 C — 40 N. *æquatur* 192. *& eſt* 1 N. 8.

V.

Si D planum in A
— B ter in A quadratum.
— A cubo
} æquetur Z ſolido.

A + B eſto E, + D planũ.
+ B quadrato ter.
— E cubo. } in E } æquabitur { Z ſolido.
+ B cubo bis.
+ D plano in B.

100 N. — 30 Q. — 1 C. *æquatur* 72. *& eſt* 1 N. 2.
400 N. — 1 C. *æquatur* 3,072. *& eſt* 1 N. 12.

VI.

Si B ter in A quadratum
+ D plano in A
— A cubo.
} æquetur Z ſolido.

A — B eſto E, { D planum.
+ B quadrato ter
— E cubo } in E. } æquabitur { Z ſolido.
— D plano in B
— B cubo bis.

18 Q. + 92 N. — 1 C. *æquatur* 1,720. *& eſt* 1 N 10.
200 N. — 1 C. *æquatur* 736. *& eſt* 1 N. 4.

vel B — A eſto E, { D planum
+ B quadrato ter.
— E cubo. } in E } æquabitur { B cubo bis.
+ D pl. in B.
— Z ſolido.

30 Q. + 100 N. — 1 C *æquatur* 1,464. *& eſt* 1 N. 6.
400 N — 1 C *æquatur* 1,536. *& eſt* 1 N. 4.

VII.

Si B ter in A quadratum.
— D plano in A.
— A cubo
} æquetur Z ſolido.

A — B eſto E, { B quadratũ ter
— D plano
— E cubo. } in E } ęquabitur { Z ſolido
+ D plano in B.
— B cubo bis.

18 Q. — 78 N. — 1 C *æquatur* 20. *& eſt* 1 N. 10.
30 Q. — 1 C. *æquabitur* 56. *& eſt* 1 N. 4.

12 Q — 18 N. — 1 C *æquatur* 20. *& eſt* 1 N. 10.
1 C — 30 N. *æquatur* 36. *& eſt* 1 N. 6.

vel B — A eſto E. — $\left.\begin{array}{l}\text{B quadratum ter.}\\ \text{— D plano.}\\ \text{— E cubo.}\end{array}\right\}$ in E } æquabitur $\left\{\begin{array}{l}\text{B cubo bis}\\ \text{— D plan. in B.}\\ \text{— Z ſolido}\end{array}\right.$

30 Q — 100 *N.* — 1 C. *æquatur* 264. *et eſt* 1 *N.* 6.
100 *N.* — 1 C *æquatur* 736. *et eſt* 1 *N.* 4.

Vt autem expurgatione per Vncias eaſque ſimplices, liberantur æquationes adfectione ſub gradu qui in Climacticorum ordine Poteſtatem proxime ſubſequitur, ſic interdum per Vncias triangulares & Pyramidales & ex ijs compoſitas, poſſunt æquationes liberari adfectione ſub gradibus inferioribus reliquis, conſideratâ coefficiente ſub graduali, vt Poteſtate, ſuæque Radicis tot Vncias ad ſumendo, quot requirit dicta Geneſis Poteſtatum purarum à Binomia Radice. vt enim coefficientes Radici de qua quæritur homogeneæ, per Vncias ſimplices taxantur, ſic homogeneæ Radicis quadrato, per Vncias Triangulares, cubo, per Pyramidales, & eo continuo ordine.

Proponatur A cubus adfectus ſub A & B quadrato, ſumetur ad expurgationem triens B.

Et ſi proponatur A quadrato-quadratum adfectum ſub A quadrato & B quadrato, ſumetur ad expurgationem illius adfectionis ſextans B.

Et ſi proponatur A quadrato-cubus adfectus ſub A cubo & B quadrato, ſumetur ad expurgationem decima pars B. nempe ſunt numeri triangulares. 3. 6. 10. 15.

Proponatur autem A quadrato-quadratum adfectum ſub A, & B cubo, ſumetur ad expurgationem illius adfectionis quadrans B.

Et ſi proponatur A quadrato-cubus, adfectus ſub A quadrato & B cubo, ſumetur ad expurgationem decima pars B. nempe ſunt numeri Pyramidales 4. 10. 20. 35.

Quæ quemadmodum ad vlteriora poſſint aptari, ſatis ſit manifeſtum. valet autem illa vnciarum Triangularium, Pyramidalium & ex ijs compoſitarum expurgatio, dum illa ea adfectione qua liberanda proponitur, obruitur æqualitas: in eius autem adfectionis deletæ locum, ſuccedunt adfectiones ſingulæ ſub aliis gradibus reliquis.

De Tranſmutatione Πρώτον-ἔσχατον, *quæ Remedium eſt aduerſus vicium negationis.*

CAP. II.

ÆQuationes in quibus homogenea adfectionum validiora negantur de Poteſtate, vtiliter per Tranſmutationem Πρώτον-ἔσχατον, emendantur: ea fit per Analogiam rationis implicitæ, applicando homogeneum comparationis ad ipſam Radicem de qua quęritur, vnde oritur incerta alia Radix, ſub cuius ſpecie, æquationem primum propoſitam liceat dirigere, & nouam ordinare.

Sic enim adfectiones illic negatæ, tranſeunt hic in affirmatas, & e contra, ſalua numerorum Symmetria. prodeſt etiam interdum ad Aſymmetrias.

Nomen autem Πρώτον-ἔσχατον ſortita eſt, ab eo ad quem propoſita primum æquatio reuocatur, Analogiſmo: quoniam in eius formula, terminus de quo primum quęrebatur, eſt primus, is qui poſt metamorphoſin primus, fit poſtremus, vel e conuerſo. quæ quoniam ex ipſâ operis formula, facilius percipiuntur.

Proponatur $\left.\begin{array}{l} A\ \text{cubus} \\ -B\ \text{plano in}\ A \end{array}\right\}$ æquari z ſolido.

Sit autem explicanda æqualitas.

Quoniam igitur z ſolidum, Poteſtas eſt adfecta negate, de negatis autem ars non ſtatuitur, quæ proponitur æquatio in explicabilem primum tranſmutanda eſt, qualis quæ adficietur adfirmate, quo circa $\frac{Z\ \text{ſolidum}}{A}$ eſto E planum, ergo $\frac{Z\ \text{ſolidum.}}{E\ \text{plano.}}$ erit A, vnde ex iis quæ proponuntur.

$$\left.\begin{array}{l} \frac{Z\ \text{ſolido-ſolido-ſolidum}}{E\ \text{plano-plano-plano.}} \\ -\frac{B\ \text{plano in}\ Z\ \text{ſolidum}}{E\ \text{plano}} \end{array}\right\} \text{æquabitur}\ Z\ \text{ſolido.}$$

Ducantur, omnia in E plano-plano-planum.

$$\left.\begin{array}{l} Z\ \text{ſolido-ſolido-ſolidum} \\ -B\ \text{plano in}\ Z\ \text{ſolidum in}\ E\ \text{plano-planum.} \end{array}\right\} \text{æquab.}\ z\ \text{ſolid. in}\ E\ \text{pl. pl. pl.}$$

& omnibus per z ſolidum diuiſis, adhibitaq; congruâ Antitheſi,

$$\left.\begin{array}{l}\text{E plano plano-planum.}\\ +\ \text{3 plano in E plano-planum.}\end{array}\right\}\ \text{æquabitur Z ſolido-ſolido.}$$

Aliter.

$$\left.\begin{array}{l}\text{E plani cubus}\\ +\ \text{B plano in E plani quadratum}\end{array}\right\}\ \text{æquabitur Z ſolidi quadrato.}$$

quæ æquatio omnino explicabilis eſt, ex tradita Analyſi poteſtatum cubicarum, quæ adficiuntur ſub datâ coefficiente adfirmate. Innoteſcit igitur E planum, ad quod quum applicabitur Z ſolidum, exurget A de qua primum quærebatur.

Analogiſmus autem æqualitatis de A enunciatiuus, erat

Vt A ad l c. Z ſolidi, ita l c. Z ſolido-ſolidi ad $\begin{array}{l}\text{A quadratum}\\ -\ \text{B plano.}\end{array}$

hoc eſt ad E planũ, quo pertinet nominis Πρώτον-ἔσχατον notatio.

Proponatur 1 C — 96 N. æquari 40.

Effecta $\frac{40}{1 N.}$ *eſſe 1 N. 1 C. + 96 Q. æquabitur 1,600. & fit 1 N. 4.*

Quare $\frac{40}{4}$ *ſeu 10. eſt Radix primum quæſita.*

Et ſi accidat in propoſita primum æquatione cubica, homogeneum comparationis eſſe in ſua ſoliditate irrotionale, ſed facultate velut quadratica rationale, ea Aſymmetria in æquatione noua euaneſcet.

Proponatur 1 C — 10 N. æquari l 48. effecta $\frac{l\ 48.}{1 N.}$ *eſſe 1 N.*

1 C + 10 Q æquabitur 48. & fiet 1 N. 2.

Quare $\frac{l\ 48}{l\ 4.}$ *ſeu l 12. eſt Radix primum quæſita.*

$$\text{Rurſus proponatur}\ \left.\begin{array}{l}\text{A quadrato-quadratum.}\\ -\ \text{B in A cubum}\\ -\ \text{D plano in A quadratum.}\end{array}\right\}\ \text{æquari Z pl. plano.}$$

Sit autem explicanda æqualitas.

Quoniam igitur Z plano-planum eſt Poteſtas adfecta negate, de negatis autem ars non ſtatuitur, quæ proponitur æquatio in explicabilem tranſmutanda eſt: quocirca

$\frac{\text{Z plano-planum.}}{\text{A}}$ eſto E ſolidum. ergo $\frac{\text{Z plano-planum.}}{\text{E ſolido.}}$ erit A.

Itaque ex iis quæ proponuntur.

$$\left.\begin{array}{l}\dfrac{\text{Z plano-plano-plano-plano-plano-plano-plano-planum.}}{\text{E ſolido-ſolido-ſolido-ſolido.}}\\ -\ \dfrac{\text{B in Z plano plano plano plano-plano-planũ.}}{\text{E ſolido ſolido-ſolido.}}\\ -\ \dfrac{\text{D plano in Z plano-plano-plano-planũ.}}{\text{E ſolido-ſolido.}}\end{array}\right\}\ \text{æquab. Z pl. plano}$$

Ducantur omnia in E ſolido-ſolido-ſolido-ſolidum.

Z plano-plano-plano-plano-plano-plano-plano-planum.
—B in z plano-plano-plano-plano-plano-planum in E ſolidum
—D plano in Z plano-plano-plano-planum in E ſolido-ſolidum.

æquabitur.

Z plano-plano in E ſolido-ſolido-ſolido-ſolidum.

Et omnibus per z plano planum diuiſis, adhibitaque Antitheſi.

z ſolidi quadrato-quadratum.
—D plano in z plano-planum in E ſolidi quadratum.
+B in z plano-plano-plano-planum in E ſolidum.

æquabitur z plano-plani cubo.

Proponatur 1QQ—3C.—8Q. *æquari* 50.
Effecta $\frac{50}{1N}$ *eſſe* 1N.

1QQ.+400Q+7,500,N. *æquabitur* 125,000. *&* *fiet* 1 *N*. 10.
Quare $\frac{50}{10}$ *ſeu* 5, *eſt Radix primum quæſita.*

Et ſi accidat in propoſita primum æquatione quadrato-quadratica, homogeneum comparationis eſſe in ſuâ plano-planitie irrationale, ſed facultate veluti cubicâ rationale, ea Aſymmetria in æquatione nouâ euaneſcit.

Proponatur 1QQ.—8N. *æquari* l c. 80.
Effecta $\frac{1\,c.\,80.}{1N.}$ *eſſe* 1N.

1QQ+8c. *æquabitur* 80. *&* *fiet* 1 *N*. 2.
Quare $\frac{l\,c\,80.}{l\,c\,8.}$ *ſeu* l c 10. *eſt Radix primum quæſita.*

Tranſmutationis Πρῶτον-ἔσχατον, opus dum retexitur in has prorſus recidit, ideo retinendas & ordinandas præceptiones.

I.

In quadratis affectis, homogeneum comparationis inuariatum conſiſtito: in cubis ducitor quadratice: in quadrato-quadratis, cubice: in quadrato-cubis, quadrato-quadratice: in cubo-cubis, quadrato cubicè: & ita deinceps.

II.

Homogenea ſub gradibus adficientia negate, in homogenea ſub gradibus reciprocis adficientia affirmate tranſeunto: & è contrâ, adficientia adfirmate in afficientia negate.

III.

Coefficiens adfectionis ſub latere, inuariata conſiſtat: coefficiens adfectionis ſub quadrato, ducitor in homogeneum comparationis.

Coefficiens adfectionis ſub cubo, in homogenei comparationis quadratum.

Coefficiens adfectionis ſub quadrato-quadrato, in homogenei comparationis cubum.

Coefficiens adfectionis ſub quadrato-cubo, in homogenei cōparationis quadrato-quadratum. & ita deinceps.

IV.

Atque ad cognitam Radicem Poteſtatis ita nouiter adæquatæ, dum applicabitur homogeneum comparationis pertinens ad æquationem quæ proponebatur, reſtituere latus de quo primum quærebatur pronunciato.

Plane quæcunque magnitudo ad nouam quærendam Radicem applicetur, ita vt ex ea applicatione oriatur Radix de qua primum quęritur, ſi ſub ea ſpecie dirigatur propoſita æquatio, & ſecundum artem tranſmutetur, ſemper adfectiones quæ erant negatæ, tranſibunt in adfirmatas ſaluâ numerorum Symmetria: ſed ideo conuenientior eſt applicatio homogenei comparationis, ne forte accidens fractionis nouam rurſus exigat reductionem in Arithmeticis, at in Geometricis, fœlicius applicatur magnitudo homogenea quadrato Radicis de qua quæritur.

Proponatur A cubus — B in A quadratum. } æquari B in z quadratum.

$\frac{B \text{ in } z}{E}$ eſto A. → E cubus + B quadrato in E. } æquabitur B quadrato in z.

De Anaſtrophê.

Aut quemadmodum aduerſus vocum Ἀμφιβολίας *in æquationibus correlatis, ex data Radice vnius habetur alterius notitia.*

CAP. III.

ANaſtrophe eſt æquationum inuersè negatarum in ſuas correlatas tranſmutatio, ita inſtituta, vt quæ prima proponitur æquatio, ea ope ſuæ correlatæ per irregularem Climacticum deſcenſum, reducatur ad depreſſiorem, ideoque magis explicabilem. pertinet ad vitandum, tanquam in æqualitatibus inuersé negatis accidere diximus, Amphiboliã,& dyſmechanian, in cubicis, quad. cubicis & exinde per binos alternos gradus Climacticis. quod enim ad quadraticas, quadrato-quadraticas, & exinde per binos alternos gradus Climacticos attinet, earum reductioni non proficit Anaſtrophe, verum recurrendum eſt ad Climacticam Parapleroſin, de qua dicetur ſuo loco.

Anaſtrophes opus ita perficitur: primum Poteſtati Radicis de qua quęritur, adiicitur Poteſtas Radicis æque-alta, talium enim Poteſtatum aggregatum, commode diuiſionem recipit in prædicto Climactericarum ordine. deinde homogeneum comparationis, vna cum tranſlatis adfectionum homogeneis & addita Poteſtate, comparatur effictæ magnitudini, quæ eandem diuiſionem recipiat: quâ comparatione inciditur in æqualitatem correlatam, vel affirmatam, vel negatam directè. cęterum addititiæ Poteſtatis Radice cognitâ, oriundæ magnitudines ex vna parte, oriundis magnitudinibus ex altera comparantur commode, & æqualitas alioquin δυσμηχάνη, per medium εὐμηχανωτέρης, deprimitur in εὐμηχάνισας: quod vt exemplis fiat apertius, primum ad Anaſtrophen Cubicarum.

PROBLEMA. I.

PRoponatur $\left.\begin{array}{l}\text{B planum in A}\\ -\text{A cubo}\end{array}\right\}$ æquari z ſolido.

Quoniam igitur de ſolido negatur cubus, anceps æquatio eſt, neque ad Analyſin idonea: vitanda igitur ambiguitas eſt & dyſmechania: quocirca fiat Anaſtrophe, atque idcirco ex iis quæ proponuntur, adhibita metatheſi, A cubus æquatur $\left\{\begin{array}{l}\text{B pl. in A}\\ -\text{Z ſol.}\end{array}\right.$

Vtrique parti addatur E cub. ergo $\left.\begin{array}{l}\text{A cubus}\\ +\text{E cubo.}\end{array}\right\}$ æquabitur $\left\{\begin{array}{l}\text{B pl in A}\\ +\text{E cubo.}\\ -\text{Z ſol.}\end{array}\right.$

Prima autem æquationis pars commode diuiſionem recipit ab A + E, ad aggregatum enim laterum, dum applicatur aggregatum cuborum, oritur aggregatum quadratorum, minus rectangulo à lateribus, quare orietur ex ea applicatione $\begin{array}{l}\text{A quadratum}\\ -\text{E in A}\\ +\text{E quadrato.}\end{array}$

Reſtat igitur vt altera æqualitatis pars, applicata ad A + E, faciat aliquod datum planum iam ortis reliquis comparandum, id autem commode fiet, ſi in locum E cubi minus Z ſolido, ſubſtituetur B planum in E, naſcetur enim B planum. Æquiualeat igitur vt ſubſtituatur: quoniam igitur E cubus minus z ſolido, adæquatur ex hypotheſi B plano in E, ergo per Antitheſin E cubus minus B plano in E, æquabitur z ſolido.

Itaque $\left.\begin{array}{l}\text{A quadratum.}\\ -\text{E in A}\\ +\text{E quadrato.}\end{array}\right\}$ æquabitur B plano.

Nota igitur ſit E ex Analyſi, adhibitâ per antecedens Caput decenti præparatione, vt pote eſto illa D, ergo.

$\left.\begin{array}{l}\text{A quadratum}\\ -\text{A in D}\\ +\text{D quadrato.}\end{array}\right\}$ æquabitur B plano.

& ordinando $\left.\begin{array}{l}\text{D in A}\\ -\text{A quadrato}\end{array}\right\}$ æquabitur $\left\{\begin{array}{l}\text{D quadrato}\\ -\text{D plano.}\end{array}\right.$

Æquatio igitur inuerſè negata, in negatam directe, eiuſdem ordinis tranſmutata eſt, ita vt data Radice nouæ æquationis, fiat Climacticus irregularis deſcenſus in depreſſiorem, de Radice primum propoſitâ explicabilem: quod opus dicitur Anaſtrophe, & faciendum erat.

Hinc ordinatur Theorema.

THEO-

THEOREMA. I.

Si B planum in A
— A cubo } æquatur z ſolido.

igitur E cubus
— B plano in E } æquabitur Z ſolido.

E autem innoteſcat eſſe D: D in A
— A quadrato. } æquabitur { D quad.
— B plano.

39 N. — 1 C. æquatur 70.

Igitur 1 C — 39 N. æquabitur 70, & ſit tum 1 N. 7. itaq; 7 N. — 1 Q. æquabitur 10. & iſta eſt Radix primum quæſita, & ſit 2 vel 5.

PROBLEMA. II.

Sed proponatur B in A quadratum
— A cubo. } æquari z ſolido.

Oportet Anaſtrophen facere.

Ex iis igitur quæ proponuntur, adhibita congrua Metatheſi.

A cubus æquabitur, B in A quadratum
— Z ſolido.

& vtrique æqualitatis parti addendo E cubum.

A cubus
+ E cubo. } æquabitur { B in A quadratum
+ E cubo
— Z ſolido.

At vero z ſolidum minus E cubo, æquetur B in E quadratum: id eſt E cubus plus B in E quadratum, æquetur z ſolido.

A cubus
+ E cubo. } æquabitur { B in A quadratum.
— B in E quadratum

Conſequenter oportet ſolidum adfectum adiunctione Cubi reducere: omnia ducantur per A + E. igitur,

A quadratum.
— E in A.
+ E quadrato. } æquabitur B in A — E.

Innoteſcat autem E eſſe D ex Analyſi, atque adeo ordinetur quadratica æquatio. Ergo

B in A
+ D in A
— A quadrato. } æquabitur { D quadrato.
+ B in D

Æquatio igitur inuerſe negata, in adfirmatam eiuſdem ordinis transſmutata eſt, ita vt data Radice nouæ æquationis, fiat irregularis deſcenſus, in depreſſiorem, quod opus dicitur Anaſtrophe, & faciendum erat.

Hinc ordinatur Theorema.

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THEOREMA. II.

Si B in A quadratum
— A cubo } æquetur z ſolido.

B in E quadratum
+ A cubo } æquetur z ſolido.

E autem innoteſcat eſſe D, + D
B } in A
— A quadrato } æquabitur { B in D
+ D quad.

7 Q. — 1 C. æquatur 36. igitur
7 Q. + 1 C. æquatur 36. & fit hic 1N. 2. quare
9 N. — 1 Q. æquabitur 18. & iſta 1 N. eſt Radix primum quæſita.
& fit 3. vel 6.

Secundo ad Anaſtrophen quadrato-cubicarum.

PROBLEMA. III.

PRoponatur B plano-planum in A
— A quadrato cubo. } æquari Z plano-ſolido
Oportet Anaſtrophen facere.

Ab E — A effingatur quadrato-quadratum, ſingularia efficta plano-plana ſimpliciter ſumpta nec repetita, erunt

E quadrato-quadratum
— A in E cubum
+ A quadrato-in E quadratum
— A cubo in E
+ A quadrato quadrato.

Ducantur in E + A, fit E quadrato-cubus
+ A quadrato-cubo.

At ex iis quæ proponuntur, A quad. cubus valet { B pl. planũ in A
— Z pl. ſolido.

quare E quadrato-cubus.
+ A quadrato-cubo. } æquabitur { E quadrato-cubo.
+ B plano-plano in A
— Z plano-ſolido.

Vtraque pars applicetur ad E + A. Ex prima igitur æquationis parte oritur,

E quadrato-quadratum
— A in E cubum
+ A quadrato-in E quadratum
— A cubo in E
+ A quadrato-quadrato. } vt ratio compoſitionis indicat.

At quid oriatur ex ſecunda non liquet: ſed ſane comparetur tali plano-ſolido, vt plano-planum ortiuum non poſſit ignorari.

$$\text{Quare}\left\{\begin{array}{l}\text{B plano-planum in E}\\ +\text{B plano-plano in A}\end{array}\right\}\text{æquetur alteri eius parti,}$$

$$\text{videlicet,}\left\{\begin{array}{l}\text{E quadrato cubo.}\\ +\text{B plano-plano in A}\\ -\text{Z plano-ſolido.}\end{array}\right.$$

Ergo deleta vtrinque affectione ſub A gradu, & factâ congruâ Antitheſi.

$$\left.\begin{array}{l}\text{E quadrato cubus}\\ -\text{B plano plano in E}\end{array}\right\}\text{æquabitur z plano-ſolido.}$$

Et quum innoteſcet E eſſe D, pronunciabitur

$$\left.\begin{array}{l}\text{D quadrato quadratum.}\\ -\text{D cubo in A}\\ +\text{D quadrato in A quadratum}\\ -\text{D in A cubum}\\ +\text{A quadrato-quadrato.}\end{array}\right\}\text{æquari B plano-plano.}$$

& æqualitate ſecundum artem ordinatâ.

$$\left.\begin{array}{l}\text{D cubum in A}\\ -\text{D quadrato in A quadratum}\\ +\text{D in A cubum.}\\ -\text{A quadrato quadrato.}\end{array}\right\}\text{æquari}\left\{\begin{array}{l}\text{D quadrato-quadrato}\\ -\text{B plano-plano.}\end{array}\right.$$

Facta eſt igitur Anaſtrophe ſicut Imperabatur.

Hinc ordinatur Theorema.

THEOREMA. III.

$$\text{Si}\left.\begin{array}{l}\text{B plano-planum in A}\\ -\text{A quadrato-cubo.}\end{array}\right\}\text{æquetur Z plano-ſolido.}$$

$$\left.\begin{array}{l}\text{E quadrato-cubus}\\ -\text{B plano plano in E}\end{array}\right\}\text{æquetur z plano-ſolido.}$$

$$\text{\& E innoteſcat eſſe D,}\left.\begin{array}{l}\text{D cubus in A}\\ -\text{D quadrato in A quadratum}\\ +\text{D in A cubum}\\ -\text{A quadrato-quadrato.}\end{array}\right\}\text{æqu.}\left\{\begin{array}{l}\text{D qu.q.}\\ -\text{B pl.pl.}\end{array}\right.$$

11 N. — 1 QC. *æquetur* 10.

Igitur 1 QC — 11 N. *æquabitur* 10. *& hic fit* 1 *N.* 2.

$$\text{\textit{Quare}}\left.\begin{array}{l}\text{8 N}\\ -\text{4 Q.}\\ +\text{2 C}\\ -\text{1 QQ.}\end{array}\right\}\text{\textit{æqu. 5. \& iſta 1 N. eſt Radix primum quæſita, \& fit 1.}}$$

PROBLEMA. IIII.

$$\text{Si proponatur}\left.\begin{array}{l}\text{B in A quadrato-quadratum.}\\ -\text{A quadrato-cubo.}\end{array}\right\}\text{æquari z plano-ſolido.}$$

Ergo per congruam metatheſin, & E quadrato-cubi communem adiunctionem,

A quadrato-cubus
+ E quadrato-cubo. } æquatur { B in A quadrato quadratum
— Z plano-ſolido.
+ E quadrato-cubo.

Vtraq; pars applicetur ad A + E; illic oritur { A quad.quad.
— A in E cubum.
+ A quad. in E quad.
— A cubo in E
+ A quadrato-quad.

Quod ſi B in A quadrato-quadratum.
— B in E quadrato-quadratum. } ęquetur alteri parti.

Ea quoque commodam diuiſionem recipiet ab A + E, orietur enim,

B in A cubum
— B in A quadratum in E
+ B in E quadratum in A
— B in E cubum.

Adæquetur ergo, & ea pro pter vt ęqualitas rite ordinetur.

E quadrato-cubus.
+ B in E quadrato-quadratum. } ſtatuitor z plano-ſolido æquale.

& E innoteſcat eſſe D. tādē igitur. D quad.qu.
— D cub. in A
+ D qu. in A qu.
— D in A cub.
+ A quad.qu. } æq. { B in A cub.
— B in A q. in D
+ B in D q. in A
— B in D cu.

& omnia ordinando, B in D quadratum in A
+ D cubo in A
— B in D in A quadratum
— D quadrato in A quadratum.
+ B in A cubum
+ D in A cubum.
— A quadrato quadrato. } æqu. { B in D c.
+ D qu.q.

Itaque facta eſt Anaſtrophe, ſicut imperabatur.
Hinc ordinatur Theorema.

THEOREMA. IIII.

Si B in A quadrato-quadratum
— A quadrato-cubo } æquetur Z plano ſolido.

E quadrato cubus
+ B in E quadrato-quadratum. } æquatur z plano-ſolido.

& E innoteſcat eſſe D, B in D quad.
+ D cubo. } in A
— B in D
— D quad. } in A quad.
+ B
+ D } in A cubum.
— A quadrato-quadrato } æquab. { B in D cu.
+ D qu.qu.

11 QQ. — 1 QC. æquatur 10,000.

Igitur 11 QQ. + 1 QC. æquabitur 10,000. & fit 1 N, 5.

Quare
400 N.
— 80 Q.
+ 16 C.
— 1 QQ. } æquabitur 2,000.

& iſta 1 N. eſt Radix primum quæſita, & fit 10.

Vſurpatur quoque Anaſtrophe contraria interdum via, vt quum in æquatione ambigua, contingit vnam Radicum dari è duabus pluribuſue, de quibus æquatio poteſt explicari: repetuntur Anaſtrophes veſtigia ad aſſequendum Radicem correlatæ, variaque fluunt inde & ordinantur Theoremata, qualia ſunt in cubis.

THEOREMA. V.

Si A cubus — B plano in A. } æquetur Z ſolido.
& rurſus B planum in E — E cubo } æquetur Z ſolido.

Innoteſcat autem E eſſe D.

A quadratum — D in A } æquabitur { B plano — D quadrato.

Quoniam enim A cubus — B plano in A. } æquatur Z ſolido.
Ergo A cubus — B plano in A. } æquab. { B plano in D — D cubo.
& per metatheſin. A cubus. + D cubo } æquabitur { B plano in A. + B plano in D.

Vtraq; pars æquationis diuiditor per A + D, fit
A quad.
+ D quad.
— D in A. } æqu. B pl.

Qua æquatione ſecundum artem concepta.
A quadratum. — D in A } æquatur { B plano — D quadrato.
Vt eſt ordinatum.

Si 1 C. — 8 N. æquetur 7.

Igitur 8 N. — 1 C æquabitur 7. & quoniam 1 N. poteſt eſſe 1.

1 Q — 1 N æquabitur 7. vnde Radix primum quæſita fit L. 29/4 + 1/2.

THEOREMA. VI.

Si $\left.\begin{array}{l}\text{B in A quadratum}\\ +\text{A cubo.}\end{array}\right\}$ æquetur z ſolido.

& rurſus $\left.\begin{array}{l}\text{B in E quadratum}\\ -\text{E cubo.}\end{array}\right\}$ æquetur z ſolido.

Innoteſcat autem E eſſe D, $\left.\begin{array}{l}\text{A quadratum}\\ +\text{B} - \text{D in A}\end{array}\right\}$ æquabitur $\left\{\begin{array}{l}\text{B in D}\\ -\text{D quadr.}\end{array}\right.$

Quoniam enim $\left.\begin{array}{l}\text{B in A quadratum}\\ +\text{A cubo.}\end{array}\right\}$ æquatur z ſolido.

& rurſus $\left.\begin{array}{l}\text{B in D quadratum}\\ -\text{D cubo.}\end{array}\right\}$ æquatur z ſolido.

Ergo $\left.\begin{array}{l}\text{B in A quadratum}\\ +\text{A cubo.}\end{array}\right\}$ æquabitur $\left\{\begin{array}{l}\text{B in D quadratum}\\ -\text{D cubo.}\end{array}\right.$

& per metatheſin $\left.\begin{array}{l}\text{D cubus}\\ +\text{A cubo.}\end{array}\right\}$ æquabitur B in $\left\{\begin{array}{l}\text{D quadratum}\\ -\text{A quadrato.}\end{array}\right.$

vtraq; pars æquationis diuiditor per D + A, fit $\left.\begin{array}{l}\text{D quad.}\\ +\text{A quad.}\\ -\text{D in A.}\end{array}\right\}$ æq. $\left.\begin{array}{l}\\ \text{B in}\end{array}\right\}\begin{array}{l}\text{D}\\ -\text{A}\end{array}$

qua ęquatione ſecũdũ artẽ conceptâ, $\left.\begin{array}{l}\text{A quad.}\\ +\text{B} - \text{D in A}\end{array}\right\}$ æqu. $\left\{\begin{array}{l}\text{B in D}\\ -\text{D qu.}\end{array}\right.$ vt eſt ordinatum.

Si 9 Q. + 1 C æquetur 8.

Igitur 9 Q — 1 C æquabitur 8. & quoniam 1 N. poteſt eſſe 1.

1 Q + 8 N. æquatur 8. unde Radix primum quæſita fit l 24 — 4.

Quibus Theorematis finitima ſunt ea, quibus ex data vna ambiguarum Radicum, habetur alterius comitis notitia: nempe.

THEOREMA. VII.

Si $\left.\begin{array}{l}\text{B planum in A}\\ -\text{A cubo.}\end{array}\right\}$ æquetur z ſolido.

& rurſus $\left.\begin{array}{l}\text{B planum in E}\\ -\text{E cubo.}\end{array}\right\}$ æquetur z ſolido.

Innoteſcat autem E eſſe D, $\left.\begin{array}{l}\text{A quad.}\\ +\text{D in A.}\end{array}\right\}$ æquabitur $\left\{\begin{array}{l}\text{B plano}\\ -\text{D quadrato.}\end{array}\right.$

Quoniam enim $\left.\begin{array}{l}\text{B planum in A}\\ -\text{A cubo.}\end{array}\right\}$ æquatur z ſolido.

& rurſus $\begin{array}{l}\text{B planum in D}\\ -\text{D cubo.}\end{array}$ æquatur z ſolido.

igitur $\left.\begin{array}{l}\text{B planum in A}\\ -\text{A cubo}\end{array}\right\}$ æquab. $\left\{\begin{array}{l}\text{B plano in D}\\ -\text{D cubo.}\end{array}\right.$

& per metatheſin. $\left.\begin{array}{l}\text{A cubus.}\\ -\text{D cubo}\end{array}\right\}$ æquabitur $\left\{\begin{array}{l}\text{B plano in A.}\\ -\text{B plano in D.}\end{array}\right.$

& vtraq; æquationis parte per A — D diuiſa, fit $\left.\begin{array}{l}\text{A quad.}\\ +\text{D quad.}\\ +\text{D in A.}\end{array}\right\}$ æqu. B pl.

Qua ęquatione ſecundum artem concepta.

$$\left.\begin{array}{l}\text{A quadratum.}\\ +\text{D in A}\end{array}\right\} \text{æquatur} \left\{\begin{array}{l}\text{B plano}\\ -\text{D quadrato.}\end{array}\right.$$

Si 8 N. — 1 C. æquetur 7. poteſt 1 N. eſſe 1.

Quare 1 Q + 1 N. æquatur 7. & rurſus fit 1 N. l. $\frac{19}{4}$ — $\frac{1}{2}$

THEOREMA. VIII.

Si $\left.\begin{array}{l}\text{B in A quad.}\\ -\text{A cubo}\end{array}\right\}$ æquetur Z ſolido.

& rurſus $\left.\begin{array}{l}\text{B in E quadr.}\\ -\text{E cubo}\end{array}\right\}$ æquetur Z ſolido.

Innoteſcat autem E eſſe D: $\left.\begin{array}{l}\text{A quadratũ.}\\ \left.\begin{array}{l}+\text{D}\\ -\text{B}\end{array}\right\}\text{in A}\end{array}\right\}$ æquabitur $\left\{\begin{array}{l}\text{B in D}\\ -\text{D quad.}\end{array}\right.$

Quoniam enim $\left.\begin{array}{l}\text{B in A quadr.}\\ -\text{A cubo.}\end{array}\right\}$ æquatur Z ſolido.

& rurſus $\left.\begin{array}{l}\text{B in D quadratum}\\ -\text{D cubo}\end{array}\right\}$ æquatur Z ſolido.

igitur $\left.\begin{array}{l}\text{B in A quadratum}\\ -\text{A cubo}\end{array}\right\}$ ęquatur $\left\{\begin{array}{l}\text{B in D quadratum}\\ -\text{D cubo.}\end{array}\right.$

& per metatheſin $\left.\begin{array}{l}\text{A cubus}\\ -\text{D cubo.}\end{array}\right\}$ æquab. B in $\left\{\begin{array}{l}\text{A quadratum}\\ -\text{D quadrato.}\end{array}\right.$

Vtraq; pars æquationis diuiditor per A — D.

fit $\left.\begin{array}{l}\text{A quadratum}\\ +\text{D quadrato.}\\ +\text{D in A}\end{array}\right\}$ ęquale $\left\{\begin{array}{l}\text{B in A}\\ +\text{B in D}\end{array}\right.$

quâ æquatione rite conceptâ. $\left.\begin{array}{l}\text{A quadratum}\\ +\text{D in A}\\ -\text{B in A.}\end{array}\right\}$ æquabitur $\left\{\begin{array}{l}\text{B in D}\\ -\text{D q.}\end{array}\right.$

Vt eſt ordinatum.

Si 9 Q — 1 C. æquetur 8 poteſt 1 N. eſſe 1.

Quare $\left.\begin{array}{l}\text{Q}\\ -8\text{ N}\end{array}\right\}$ *æquabitur 8. & rurſus fit 1 N. l. 24 — 4.*

De Iſomeria, aduerſus vicium Fractionis.

CAP. IIII.

ISomæria eſt ſpecies trãſmutationis per multiplicationẽ, ita inſtituta, vt æqualitates à fractis numeris quib. laborant liberẽtur.

Reducuntur videlicet fractiones ad eandem denominationem, ex lege Logiſtices.

Deinde fit ductio homogenei communis denominatoris, vel ortorum ab eo graduum, in datas coefficentes, datumque homogeneum comparationis:

Radices ducãtur in coefficiẽtes longitudines, Quadrata in coefficientes planas, homogeneaue datæ menſuræ plana, Cubi in Parabolas ſolidas, homogeneaue datæ menſuræ ſolida, & eo conſtanti ordine.

Quodq; fit ex denominatore communi, & Radice æqualitatis propoſitæ, eſt Radix æqualitatis ita præparatæ.

Interdum etiam euenit, vt multiplicatione Iſomærica non opus ſit, ſed diuiſione: applicantur videlicet coefficientes longitudines ad Radices coefficientis planæ, homogeneæque datæ menſuræ Solidæ ad cubos, & eo conſtanti ordine, quodque oritur ex applicatione Radicis ad communem denominatorem, eſt Radix æqualitatis ita præparatæ: fundamentum autem ſuum habet & demonſtrationem, ex ſymbolo ęqualitatum, quo cauetur, ſi æqualia per æqualia multiplicentur vel diuidantur, facta vel orta eſſe ęqualia.

Ita enim feciſſe Iſomęriam in ſumma nihil aliud eſt, quam propoſitæ æqualitatis Poteſtatem, & adfectionum & comparationis homogenea per eundem terminum multiplicaſſe, aut diuiſiſſe. multiplicationis autem paratior vſus eſt quam diuiſionis, proponatur ſiquidem.

A cubus $+ \frac{\text{D ſolido in A}}{\text{D}}$ æquari Z ſolido.

Oportet iam æqualitatem a fractionibus quibus laborat, liberare. D in A eſto E planum, ergo $\frac{\text{E planum}}{\text{D}}$ erit, A quare.

$$\left.\frac{\text{E plani cubus} + \text{B ſolido in D in E planum.}}{\text{D cubo.}}\right\}$$ æquabitur z ſoli do.

Omnia per D cubum $\left.\begin{array}{l}\text{E plani cubus}\\ + \text{B ſolido in D in E planum.}\end{array}\right\}$ æq. z ſ. in D cu.

1 C + $\frac{1}{2}$ N, æquatur 225.

Igitur κατ' ἰσομοιρίαν 1 C + 6 N. æquabitur 1,800.

& Radix præparatæ, ad Radicem propoſitæ ſe habet vt 2. ad 1.

Itaque quum ſit hic 12, illic erit 6.

Et ſi proponatur A cubus. $+ \frac{\text{B ſolido in A}}{\text{D}}$ æquari $\frac{\text{Z plano-plano.}}{\text{D}}$

Ipſiſmet veſtigiis $\begin{array}{l}\text{E plani cubus}\\ + \text{B ſolido in D in E planum}\end{array}$ ęquab. z pl. pl. in D quad.

1 C.

1 C + $\frac{3}{2}$ N. æquatur $\frac{265}{2}$.

Igitur κατ' ἰσομοιρίαν, 1 C + 6 N. æquabitur 1,060.

& quum ſit hic 1 N. 10. illic erit 5.

Et ſi proponatur A cubus. + $\frac{\text{B plano in A quad.}}{\text{D}}$ æquari Z ſolido,

Iiſdem veſtigiis E plani cubus + B plano in E pl. quadr. æquab. Z ſ. in D cub.

1 C. + $\frac{3}{2}$ Q æquatur 270.

Igitur κατ' ἰσομοιρίαν 1 C + 3 Q. æquabitur 2,160

& quum ſit hic 1 N. 12. illic erit 6.

Et ſi proponatur A cubus + $\frac{\text{B plano in A quad.}}{\text{D}}$ æquari $\frac{\text{Z plano-plano.}}{\text{D}}$

Iiſdem veſtigiis E plani cubus + B plano in E pl. quad. } æquab. Z pl. pl. in D quad.

1 C + $\frac{3}{2}$ Q æquatur $\frac{325}{2}$

Igitur κατ' ἰσομοιρίαν 1 C. + 3 Q æquabitur 130.

& quum ſit hic 1 N. 10. illic erit 5.

Sed proponatur A cubus + $\frac{\text{B ſolido in A}}{\text{D}}$ æquari $\frac{\text{Z plano-ſolido.}}{\text{H plano.}}$

D in H planum in A, eſto E plano-pl. ergo $\frac{\text{E plano-planum.}}{\text{D in H planum.}}$ erit A.

Quare $\frac{\text{E plano-plani cubus.} + \text{D in H plano-planum in B ſolidum in E plano-planum.}}{\text{D cubo in H plano-plano planum.}}$ æquabitur $\frac{\text{Z plan o-ſolido.}}{\text{H plano}}$ Omnia per D cubum in H plano-planum.

E plano-plani cubus.
+ B ſolido in D in H plano-planum in E plano-planum.
æquabitur Z plano-ſolido in D cubum in H plano-planum.

1 C + $\frac{11}{12}$ N. æquatur. $\frac{19}{4}$

Igitur κατ' ἰσομοιρίαν 1 C. + 2,112 N. æquabitur 525,312.

& Radix præparatæ ad Radicem propoſitæ ſe habet vt 48. ad 1.

Itaque quum ſit hic 1 N. 72. illic erit. $\frac{3}{2}$

Poterit autem ad eandem fractionem quoque reduci.

1 C. + $\frac{11}{12}$ N. æquatur $\frac{57}{12}$

Vnde per opus ἰσομοιρίαν, 1 C + 132 N. æquabitur. 8,208. & Radix præparatæ, ad Radicem propoſitæ, ſe habet vt 12. ad 1. itaque

Quum ſit hic 1 N. 18. illic erit $\frac{3}{2}$, quod eſt preparatam primum æqualitatem diuiſiſſe ἰσομοιρικῶς per 4.

1 C. + 2,112 N. æquabitur 525,312. igitur Iſomæricâ diuiſione,

1 C. + $\frac{2,112}{16}$ æquabitur $\frac{525,312}{64}$ id eſt 1 C. + 132. N. æquabitur 8,208. & quum ſit illic 1 N. 72. hic erit 18. quia Radix Iſomærica diuiſionis eſt 4.

pote ipſa interdum Iſomæria, variæque tranſmutandarum æqualitatum, ex ipſamet conſtitutione per Zeteſin ediſcendæ & ordinandæ formulæ.

De Symmetrica Climactiſmo Exemplum.

PRoponatur A cubus — B pl. in A } æquari l. Z ſolido-ſolidi.

Oportet eam æqualitatem Aſymmetria expurgare.

Quoniam igitur Aſymmetria eſt in planitie, quadrentur omnia igitur
A cubo-cubus.
+ B plano plano in A quad.
— B plano in A quad. quadratum bis. } æquatur Z ſolido-ſolido.

Ergo factum eſt quod oportuit.

1 C — 2 N. æquatur l. 1, 200.

Igitur, 1 C + 4 N. — 4 Q. æquabitur 1, 200. & fit 1 N. 12. quadratum Radicis de qua quæritur.

Poterat autem reduci ad Symmetriam per Πρῶτον ἔσχατον.

1 C. — 2 N. æquatur l, 1 2 0 0.

Igitur 1 C. + 2 Q. æquatur 1, 200. & fit 1 N. 10. unde Radix propoſita eſt l. 12.

Aliud.

Proponatur A cubus — l. c. B ſolido-ſolidi in A } æquari Z ſolido.

Oportet eam æqualitatem Aſymmetria expurgare.

Quoniam igitur Aſymmetria eſt in ſoliditate, per ea autem quæ proponuntur adhibita metatheſi, & per A diuiſione factâ:

A cubus
— Z ſolido.
―――――
A
} æquatur l c. B ſolido-ſolidi.

Omnia ducantur cubice.
A cubo-cubo-cubus
— Z ſolido in A cubo-cubũ ter.
+ Z ſol. ſolido in A cubum ter
— Z ſolido ſolido-ſolido.
―――――
A cubo.
} æq. B ſ. ſolid.

Et omnibus in A cubum ductis & rite ordinatis.

A cubo-cubo-cubus
— Z ſolido ter in A cubo-cubum.
+ Z ſolido-ſolido ter } in A cubum.
— B ſolido-ſolido. }
} æquabitur Z ſolido-ſolido-ſolido.

Ergo factum eſt quod oportuit.

1 C. — l c. 18. in 1 N. æquatur 6.

1 C. — 18 Q. + 90 N. æquatur 216. & fit 1 N. 12. cubus Radicis de qua quæritur.

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Quemadmodum æquationes Quadrato-quadraticæ deprimuntur ad Quadraticas, per medium Cubicarum à Radice plana.

Seu de Climactica Paraplerosi.

CAP. VI.

Atque hi quinque modi ad præparandum æqualitates quomodocunque adfectas, vt illæ tandem explicentur numero ſecundum canonica Analyſeos præcepta fere ſufficiunt : nam etſi Radices ſint Aſymmetræ, exhibebuntur eâ methodo veris proximę, accuratas autem exhibere, eſt Geometræ potius quam Arithmetici : ſępe tamen in Radicum Aſymmetriis, iuuabit Arithmeticum ea cubicarum ęquationum conſtitutio, quę tradita eſt de differentia vel aggregato mediarum, ex data differentia vel aggregato extremarum, præter rectangulum ſub medijs vel extremis: vel etiam iam tradenda doctrina de depreſſione æquationum quadrato-quadraticarum ad quadraticas, per medium cubicarum à Radice plana : poterat autem negotium abſolui ex quadrato-quadraticarum conſtitutione per Plaſma agnitâ, ſuſcepta noua zeteſi, at non minus fœliciter, ac fortaſſis etiam elegantius, per opus quod dicitur Climactica Paraplerosis, tribus quatuorue ſequentibus exemplificanda Problematis.

Omnino Climactica Paraplerosi, non etiam Anaſtrophe, reduci quadraticas, quadrato-quadraticas, cubo-cubicas, & exinde per binos gradus alternos Climacticas ęqualitates, iam ante animaduerſum eſt. Eſt autem ſpecies irregularis deſcenſus, adſumpto nempe ſupplemento, quo pertinet verbi Parapleroſeos notatio.

PROBLEMA. I.

AEquationem quadrato-quadrati adfecti ſub latere, per medium cubicæ Radicem habentis planam, ad quadraticam deprimere.

Proponatur $\left.\begin{array}{l}\text{A quad. quad.}\\ +\text{B ſolido in A}\end{array}\right\}$ æquari Z plano-plano.

Oportet facere quod imperatum eſt.

Ex ijs igitur quæ proponuntur, A quad. quad. æquab. $\begin{array}{l}\text{Z pl. plano.}\\ -\text{B ſolido in A}\end{array}$

Vtrique æqualitatis parti addatur $\begin{array}{l}\text{A quad. in E quadratum.}\\ +\text{E quad. quadrato.}\ \frac{1}{4}\end{array}$

$$\text{Igitur}\ \left.\begin{array}{l}\text{A quad. quadratum.}\\ +\text{A quad. in E quadratum}\\ +\text{E quad. quadrato.}\ \frac{1}{4}\end{array}\right\}\ \text{æquabitur}\ \left\{\begin{array}{l}\text{Z plano-plano.}\\ -\text{B ſolido in A}\\ +\text{A quad. in E quad.}\\ +\text{E quad. quad.}\ \frac{1}{4}\end{array}\right.$$

Omnia diuidantur ſubquadratice, illic orietur $\begin{array}{l}\text{A quadratum}\\ +\text{E quadrato.}\ \frac{1}{2}\end{array}$

Idcirco enim de induſtria A quad-quadrato, adiecta fuerunt in ſupplementum duo illa plano-plana A quad. in E quad. & E quad. quadrati $\frac{1}{4}$, quæ alioqui deficiebant à Canonica Geneſi quadrati, inſtitutâ à duabus Radicibus planis: quod ſi altera æqualitatis pars poſſet quoque diuidi ſubquadratice, quod oriretur foret æqualo A quad. + E quadrati $\frac{1}{2}$

Effingendum igitur quadratum à Radice plana, cui altera illa æqualitatis pars commode comparetur, vt ei tandem Radici planæ adæquetur A quadratum + E quadrato $\frac{1}{2}$

Sit igitur abs $\frac{\text{B ſolido}}{\text{E } 2}$ — E in A. ſic enim in comparatione euaneſcent adfectiones ſub A vel gradibus, & incidetur in æqualitatem de E, quo tendendum eſt.

$$\text{Effictũ igitur quad. erit}\ \left.\begin{array}{l}\frac{\text{B ſolido-ſolidum.}}{\text{E quad. } 4.}\\ +\text{E quad. in A quad}\\ -\text{B ſolido in A.}\end{array}\right\}\ \text{æquandum}\ \left\{\begin{array}{l}\text{Z pl. plano}\\ -\text{B ſol. in A}\\ +\text{E q. in A q.}\\ +\text{E qu. qu.}\ \frac{1}{4}\end{array}\right.$$

Et delectis vtrinque adfectionibus $\frac{\text{E quadrati in A quadratum.}}{- \text{B ſolido in A.}}$

Omnibuſque in E quadr. 4. ductis, $\left.\begin{array}{l}\text{E cubo-cubus.}\\ + \text{Z pl pl.4. in E quad.}\end{array}\right\}$ equa. B ſ. ſ.

Innoteſcat autem E quadratum eſſe D quadratum.

Ergo $\frac{\text{B ſolidum}}{\text{D 2.}} - \text{D in A}$, æquabitur $\begin{array}{l}\text{A quadrato}\\ + \text{D quadrato } \frac{1}{2}.\end{array}$

& ordinata ſecundum artem æqualitate $\left.\begin{array}{l}\text{A quad.}\\ + \text{D in A.}\end{array}\right\}$ æq. $\left\{\begin{array}{l}\frac{\text{B ſolido.}}{\text{D 2}}\\ - \text{D quad. } \frac{1}{2}.\end{array}\right.$

Et ſi proponatur $\left.\begin{array}{l}\text{A quad.quad.}\\ - \text{B ſolido in A}\end{array}\right\}$ æquari z plano-plano,

Radix plana effingendi quadrati ſtatuetur

$\frac{\text{B ſolidum}}{\text{E 2}} + \text{E in A}$, comparanda $\begin{array}{l}\text{A quadrato}\\ + \text{E quadrato.} \frac{1}{2}\end{array}$

Idem in æqualitate negatâ inuerſè conuertendo, & ſub contraria adfectionis nota argumentando.

$\left.\begin{array}{l}\text{A quad.quad.}\\ - \text{B ſolido in A}\end{array}\right\}$ æquari — z plano-plano.

Quod erit æqualia æqualibus auferre: cedet autem E quad. ſemis. $\frac{\text{B ſolido.}}{\text{E 2.}}$ quum alioqui præſtet in negatâ directe.

Hinc poterunt ordinari tria reductionis Theoremata.

THEOREMA. I.

Si $\left.\begin{array}{l}\text{A quadrato-quadratum}\\ + \text{B ſolido in A.}\end{array}\right\}$ æquetur z plano-plano.

$\left.\begin{array}{l}\text{E quadrati-cubus}\\ + \text{Z plano plano 4 in E quad.}\end{array}\right\}$ æquetur B ſolido-ſolido.

Innoteſcat autem E eſſe D $\left.\begin{array}{l}\text{A quad.}\\ + \text{D in A}\end{array}\right\}$ æquabitur $\begin{array}{l}\frac{\text{B ſolido.}}{\text{D 2}}\\ - \text{D quad.} \frac{1}{2}\end{array}$

THEOREMA. II.

Si $\left.\begin{array}{l}\text{A quadrato-quadratum}\\ - \text{B ſolido in A.}\end{array}\right\}$ æquetur z plano-plano.

$\left.\begin{array}{l}\text{E quadrati cubus}\\ + \text{Z plano-plano 4 in E quad.}\end{array}\right\}$ æquetur B ſolido-ſolido.

Innoteſcat autem E eſſe D, — A quadratum — D in A } æquabitur { — B ſolido / D 2 — D quadr. $\frac{1}{2}$

Conſtitutione autem tum huius tum antecedentis æquationis bene agnita, ſunt duo latera, a quibus quadrato-quadratorum differentia, applicata ad aggregatum laterum, facit B ſolidum: id eſt factum duplum ex rectangulo ſub lateribus in differentiam, adiunctum differentiæ cubo.

Differentia vero ipſorum laterum eſt E ſeu D. & fit z plano-planum, a quadrato differentiæ laterum plus rectangulo ſub lateribus in ipſum rectangulum. & A eſt latus vnum, hic maius, illic minus.

In ſerie vero quatuor continuè proportionalium, fit D differentia extremarum: B ſolidum, differentia cuborum â ſingulis alterne ſumptorum: z plano-planũ quod fit ex vtrauis extremarum in differentiam cuborum a reliquis alterne ſumptorum: & fit A prima, hic maior inter extremas, illic minor.

Sint proportionales continuè 2. *lc.*40. *lc.*200. 10:
1 *Q Q.* + 832 N. *æquatur* 1,680.
Igitur 1 Q + 8 *N. æquabitur.* 20. *& fit* 1 N. 2.
Et ſi 1 Q Q — 832 N *æquetur* 1,680.
Igitur 1 *Q* — 8 N. *æquabitur* 20. *& fit* 1 *N.* 10.
N*ota autem eſt* 8 *differentia inter* 2 *&* 10. *quoniam* 1 C. + 6,720 N. *æquatur* 692,224.
Vnde fit 1 N. 64. *quantum eſt quadratum abs* 8.

THEOREMA. III.

Si — B ſolidum in A — A quadrato-quadrato. } æquatur z plano-plano.
E quadrati cubus — Z plano plano 4 in E quad } æquetur B ſolido-ſolido.
Innoteſcat autem E eſſe D, — D in A — A quad. } ęquabitur { D quad. $\frac{1}{2}$ — B ſolido / D 2.

Conſtitutione autem æqualitatis bene agnita, eſt B ſolidum quod fit ſub aggregato quadratorum in aggregatum laterum. ſeu aliter: cubus aggregati duorum laterum, multatus facto duplo abs rectangulo ſub lateribus, in aggregatum laterum. & z plano planum factum abs quadrato aggregati laterum minus rectangulo, in

Effingendum eſt igitur quadratum à Radice planâ, & illud alteri æqualitatis parti, id eſt Z plano-plano vna cum ſuis adfectionibus comparandum & adæquandum, vt Radices quoque comparentur & adæquentur inter ſe: ſtatuitor idcirco Radix illa plana effingendi quadrati $\dfrac{\text{E planum in B}}{\text{Lv.}\left\{\begin{matrix}\text{B quad. 4.}\\ -\text{E plano 4.}\end{matrix}\right.} - \text{Lv.}\left\{\begin{matrix}\text{B quadrati}\\ -\text{E plano}\end{matrix}\right\}\text{ in A quadratum.}$

Sic enim in comparatione euaneſcent adfectiones ſub A vel gradibus, & incidetur in æqualitatem de E, quo tendendum eſt.

Effictum igitur quadratum erit
$$\frac{\text{E plano-plan. in B quad.}}{\begin{matrix}\text{B quad. 4.}\\ -\text{E plano 4.}\end{matrix}} \begin{matrix}+\text{B quad. in A quadratum}\\ -\text{E plano in A quadratum.}\\ -\text{E plano in B in A.}\end{matrix}$$

Adæquandum z plano-plano vna cum reliquis quæ illud comitantur & adficiunt expoſitis plano-planis, & deletis vtrinque adfectionibus ſub A & A quadrato.

$$\frac{\text{E plan. planum in B quad.}}{\begin{matrix}\text{B quad. 4.}\\ -\text{E plano 4.}\end{matrix}} \Big\{ \text{æquabitur} \left\{\begin{matrix}\text{Z plano-plano}\\ +\text{E plano-plano }\frac{1}{4}\end{matrix}\right.$$

Et omnibus ductis in $\begin{matrix}\text{B quadratum 4}\\ -\text{E plano 4}\end{matrix}$
E plano planum in B quad. æquabitur $\left\{\begin{matrix}\text{Z plano-plano in B quad. 4.}\\ +\text{E plano-plano in B quad.}\\ -\text{Z plano-plano in E plan. 4.}\\ -\text{E plano-plano-plano.}\end{matrix}\right.$

Et deleto vtrinque E plano-plano in B quadratum, omnibuſque rite ordinatis.

$$\begin{matrix}\text{E plani cubus.}\\ +\text{Z plano-plano 4 in E pl.}\end{matrix}\Big\} \text{ æquabitur z plano-plano in B quad. 4.}$$

Innoteſcat autem E planum eſſe D planum.

Ergo
$$\begin{matrix}\text{A quadratum}\\ +\text{B in A}\\ -\text{D plano }\frac{1}{2}\end{matrix}\Big\} \text{ æquabitur } \frac{\text{D plano in B}}{\text{Lv.}\left\{\begin{matrix}\text{B quad. 4.}\\ -\text{D plano 4.}\end{matrix}\right.} - \text{Lv.}\left\{\begin{matrix}\text{B quad.}\\ -\text{D plano.}\end{matrix}\right\}\text{ in A quad.}$$

Et æqualitate ſecundum artem ordinata.

$$\begin{matrix}\text{A quadratum}\\ +\text{B in A}\\ +\text{Lv.}\left\{\begin{matrix}\text{B quad.}\\ -\text{D plano}\end{matrix}\right\}\text{ in A q.}\end{matrix}\Big\} \text{ æquabitur D pl. }\frac{1}{2} + \frac{\text{D plano in B}}{\text{Lv.}\left\{\begin{matrix}\text{B quad. 4.}\\ -\text{D plano. 4}\end{matrix}\right.}$$

Etſi proponatur $\begin{matrix}\text{A quad. quadratum}\\ -\text{B in A cubum in bis}\end{matrix}\Big\}$ æquari z plano-plano.

Radix plana effingendi quadrati ſtatuetur.

$$\frac{\text{E planum in B}}{\text{Lv.}\left\{\begin{matrix}\text{B quad. 4.}\\ -\text{E plan. 4.}\end{matrix}\right.} - \text{Lv.}\left\{\begin{matrix}\text{B quad.}\\ -\text{E plano.}\end{matrix}\right\}\text{ in A quad. compar.}\left\{\begin{matrix}\text{A quad.}\\ +\text{B in A.}\\ -\text{E plan. }\frac{1}{2}\end{matrix}\right.$$

Etſi proponatur $\left.\begin{matrix}\text{B in A cubum bis.}\\ -\text{A quadrato-quadrato.}\end{matrix}\right\}$ æquari z plano-plano.

Licebit argumentari.

$\left.\begin{matrix}\text{A quad. quad.}\\ -\text{B in A cub. bis.}\end{matrix}\right\}$ æquari — Z plano-plano.

Quod erit auferre æqualia ab æqualibus.

Et Radix plana effingendi quadrati ſtatuetur.

$$\frac{\text{E planum in B}}{\text{Lv.}\left\{\begin{matrix}\text{B quad. 4.}\\ -\text{E plano. 4.}\end{matrix}\right.} - \text{Lv.}\left\}\begin{matrix}\text{B quad.}\\ +\text{E plano.}\end{matrix}\right\{ \text{in A q. comparanda.} \begin{matrix}\text{B in A}\\ +\text{E plano}\ \tfrac{1}{2}\\ -\text{A q.}\end{matrix}$$

Fit itaque omni caſu reductio quæ imperata eſt.

Hinc ordinabuntur tria reductionis Theoremata.

THEOREMA. I.

Si $\left.\begin{matrix}\text{A quad. quad.}\\ +\text{B in A cub.}\end{matrix}\right\}$ æquetur z plano-plano.

$\left.\begin{matrix}\text{E plani cubus.}\\ +\text{Z plano-plano 4 in E planum.}\end{matrix}\right\}$ æquetur z plano-plano in B quad. 4.

Innoteſcat autem E planum eſſe D planum.

$$\begin{matrix}\text{A q.}\\ +\text{B in A}\end{matrix} + \text{Lv.}\left\{\begin{matrix}\text{B quad.}\\ -\text{D plano.}\end{matrix}\right\} \text{in A quad.} \quad \text{æquab.} \frac{\text{D plano in B}}{\text{Lv.}\left\{\begin{matrix}\text{B quad. 4.}\\ -\text{D plano. 4}\end{matrix}\right.} + \text{D plano}.$$

THEOREMA. II.

Si $\left.\begin{matrix}\text{A quad. quad.}\\ -\text{B in A cub. bis.}\end{matrix}\right\}$ æquetur z plano-plano.

$\left.\begin{matrix}\text{E plani cubus}\\ -\text{Z plano-plano 4 in E planum}\end{matrix}\right\}$ æquetur z plano-plano in B q. 4.

Innoteſcat autem E planum eſſe D planum.

$$\begin{matrix}\text{Aq.}\\ +\text{B in A}\end{matrix} + \text{Lv.}\left\{\begin{matrix}\text{B quad.}\\ -\text{D plan.}\end{matrix}\right\} \text{in A quad.} \quad \text{æquab.} \frac{\text{D plano in B.}}{\text{Lv.}\left\{\begin{matrix}\text{B quad. 4.}\\ -\text{D plano. 4}\end{matrix}\right.} + \text{D plano}\ \tfrac{1}{2}$$

Conſtitutione autem tum huius tum antecedentis æqualitatis bene agnitâ, ſunt duo latera à quibus quadrato-quadratorum differentia applicata ad aggregatum cuborum facit B bis: quadratum verò differentię laterum multatæ ipſâ B, ablatum ex eiuſdem B quadrato, relinquet D planum, ſiue E planum: & fit z plano-planum ex applicatione cubi à D plano, ad differentiam quadruplam quadratorum a D & B. & A eſt latus vnum, hic maius, illic minus.

In ſerie verò quatuor continuè proportionalium B bis eſt dif-

ferentia illarum omnium alternè ſumptarum. Z plano-planum quod fit ab vtrauis extremarum in cubum differentiæ reliquarum trium alternè ſumptarum: & fit E planum ſiue D planum differentia ſubquadrupla, inter quadratum differentiæ omnium alternè ſumptarum, & quadratum differentiæ extremarum multatæ triplâ differentiæ mediarum.

Vnde Lv. $\left\{ \begin{array}{l} \text{B quad. 4.} \\ \text{— D plano. 4.} \end{array} \right.$ eſt differentia extremarum minus triplâ differentiæ mediarum: & quum prima intelligitur minor inter extremas illic fit A differentia trium primarum alternè ſumptarum, hic trium poſtremarum.

Sint proportionales continuè. 1. 2. 4. 8.

1 Q Q + 5 C. *æquatur* 216.

Igitur 1 Q + 3 N. *æquabitur* 18. *& fit* 1 N. 3.

Et ſi 1 Q Q — 5 C. *æquatur* 216.

Igitur 1 Q. — 3 N. *æquabitur* 18. *& fit* 1 N. 16.

Nota eſt autem 3. *quoniam* 1 C. + 8 6 4 N. *æquatur* 5, 400. *& fit* 1 N. 6. *differentia ſubquadrupla inter quadratum abs dato latere.* 5. *& quadratum abs.* 1. *vnde dignoſcitur ipſa longitudo quæ adiecta longitudini.* 5. *facit* 6. *duplum ipſius* 3.

THEOREMA. III.

Si $\left. \begin{array}{l} \text{B in A cubum bis} \\ \text{— A quad. quad.} \end{array} \right\}$ æquetur z plano-plano.

$\left. \begin{array}{l} \text{E plani cubus} \\ \text{— Z plano-plano 4 in E planum} \end{array} \right\}$ æquabitur z plano-plan. in B quad. 4.

Innoteſcat autem E planum eſſe D planum.

$+ \text{Lv.} \left\{ \begin{array}{l} \text{B in A} \\ \text{B quad.} \\ \text{+ D plano} \\ \text{— A quad.} \end{array} \right\}$ in A q. æquabitur Lv. $\begin{array}{l} \dfrac{\text{D plano in B}}{\text{B quad 4.}} + \text{D plano} \frac{1}{2} \\ \text{+ D plano 4.} \end{array}$

Conſtutione autem æqualitatis bene agnitâ, ſunt duo latera à quibus differentia quadrato-quadratorum, applicata differentiæ cuborum facit B bis: differentia vero inter quadratum aggregati laterum multati B, & ipſum B quadratum, relinquit D planum. & fit z plano-planum ex applicatione cubi à D plano, ad aggregatum quadruplum quadratorum a B plus D. & eſt A latus maius minúſue.

In ſerie quatuor continuè proportionalium, B bis eſt compoſita ex illis omnibus.

Z plano-planum quod fit ab vtrauis extremarum, in cubum compoſitæ ex tribus reliquis.

Et fit E planum ſeu D planum differentia ſubquadrupla inter quadratum aggregati extremarum, adiuncti triplo aggregati mediarum, & quadratum aggregati omnium.

Vnde Lv. $\left\{ \begin{array}{l} \text{B quad 4.} \\ + \text{D plano 4.} \end{array} \right\}$ eſt aggregatum extremarum, plus triplo aggregati mediarum, & fit A compoſita ex tribus primis ſiue ex tribus poſtremis.

Sint proportionales continuè 1. 2. 4. 8.

15 C. — 1 QQ. *æquatur* 2,744.

Igitur 21 N. — 1 Q. *æquabitur* 98 *& fit* 1 N. 7. *vel*. 14.

Nota eſt autem 21. *quoniam* 1 C. — 10,976 N. *æquatur* 617,400.

Et fit 1 N 126. *differentia ſubquadrupla inter* 225. *&* 729.

Vnde dignoſcitur l. 729. *id eſt longitudo* 27. *quæ adiecta longitudini* 15. *facit* 42. *duplum ipſius* 21.

PROBLEMA. III.

AEqualitatem Quadrato-quadrati adfecti tam ſub Latere quam Quadrato, per medium Cubicæ Radicem habentis planam, ad Quadraticam deprimere.

Proponatur $\left. \begin{array}{l} \text{A quadrato-quadratum.} \\ + \text{G plano in A quad. bis.} \\ + \text{B ſolido in A.} \end{array} \right\}$ æquari Z plano-plano.

Oportet facere quod imperatum eſt.

Sane ſi quadratum effingatur abs

$$\begin{array}{l} \text{A quad.} \\ + \text{G plano.} \\ + \text{E quad.} \frac{1}{2} \end{array} \quad \text{erit illud} \left\{ \begin{array}{l} \text{A quad. quad.} \\ + \text{G plan. plano.} \\ + \text{G quad in A qu. bis.} \\ + \text{E quad. quad.} \frac{1}{4} \\ + \text{E quad. in A quad.} \\ + \text{G plan. in E quad.} \end{array} \right.$$

Quoniam igitur ex ijs quæ propoſita ſunt, adhibita Metatheſi.

$$\left. \begin{array}{l} \text{A quadrato quadratum} \\ + \text{G plano in A qudratum bis} \end{array} \right\} \text{æquatur} \left\{ \begin{array}{l} \text{Z plano-plano} \\ - \text{B ſolido in A.} \end{array} \right.$$

Vtrique igitur æquationis parti addatur id quod deficit ab effecto à ſtatutâ Radice plana quadrato, ergo hac æqualium, æqualibus additione, rurſum pars parti æqualis erit.

Iam vtraq; pars diuiditor ſub quadratice, illic reuocatâ ad Analyſin Geneſi, orietur manifeſto $\begin{array}{l} \text{A quadratum.} \\ + \text{G plano} \\ + \text{E quad.} \frac{1}{2} \end{array}$

Quod ſi altera quoque æqualitatis pars poſſet diuidi ſub quadratice, quod orirerur, foret Radicibus illis planis ex prima parte ortiuis æquale.

Effingendum eſt igitur quadratum à Radice planâ, & illud alteri æqualitatis parti, id eſt z plano-plano vna cum ſuis adfectionibus comparandum & ad æquandum, vt Radices quoque comparentur & adæquentur inter ſe; & ſtatuatur idcirco Radix illa plana effingendi quadrati $\frac{\text{B ſolidum}}{\text{E bis.}}$ — E in A, ſic enim in comparatione euaneſcent adfectiones ſub A & gradibus, & incidetur in æqualitatem de E, quo tendendum eſt.

Effictum igitur quadratum erit.

+ $\frac{\text{B ſolido-ſolidum}}{\text{E quad. 4.}}$
+ E quad. in A quad.
— B ſolido in A.
} æquale {
Z plano-plano.
— B ſolid. in A
+ G plan. plano
+ E quad. quad. $\frac{1}{4}$
+ E quad. in A quad.
+ G plan in E quad.

Et deletis vtrinque adfectionibus ſub A & A quadrato.

$\frac{\text{B ſolido-ſolidum}}{\text{E q. 4.}}$ { æquabitur {
Z plano-plano
+ G plano-plano
+ E quad. quad. $\frac{1}{4}$
+ G plan in E quad.

Et omnibus in E quadratum 4. ductis & rite ordinatis.

E quadrati cubus
+ G plano 4 in E quadrati quad.
+ Z plano-plano 4. } in E quad.
+ G plano-plano 4 }
} æquabitur B ſolido-ſolido.

Innoteſcat autem E eſſe D. Aq. + D in A } æquabitur {
$\frac{\text{B ſolido}}{\text{D 2}}$
— G plano
+ D quadrato $\frac{1}{4}$

Et ſi proponatur A quad. quad. + G plano in A quadrat. bis — B ſolido in A } æquari z plano-plano.

Radix effingendi quadrati ſtatuetur.

$\frac{\text{B ſolidum}}{\text{E bis}}$
— E in A
{ comparanda } {
A quadrato
+ E plano
+ E quad. $\frac{1}{4}$

Et ſi proponantur A quad. quad. — G plano A quad. bis. — B ſolido in A. { æquari z plano-plano.

Radix plana effingendi quadrati ſtatuetur.

$\frac{\text{B ſolidum}}{\text{E bis.}}$
+ E in A
{ comparanda. {
A quadrato
— G plano
+ E quad. $\frac{1}{4}$

Et ſi proponatur G plan. in A quad. bis. + B ſolido in A — A quad. quad. { æquari z plano-plano.

Inuerſis adfectionum notis, Radix plana effingendi quadrati

ſtatuetur quoque $\left.\begin{array}{c}\dfrac{\text{B ſolidum}}{\text{E bis}}\\ +\text{E in A}\end{array}\right\}$ comparanda $\left\{\begin{array}{l}\text{A quadrato}\\ -\text{G plano}\\ +\text{E quadrato.} \frac{1}{2}\end{array}\right.$

Et ſi proponatur $\begin{array}{l}\text{G plan. in A quad. bis.}\\ -\text{B ſolido in A}\\ -\text{A quad. quadrato.}\end{array}$ $\Big\{$ æquari z plano-plano.

Inuerſis adfectionum notis, Radix plana effingendi quadrati ſtatuetur $\left.\begin{array}{c}\dfrac{\begin{array}{c}\text{E in A}\\ +\text{B ſolido}\end{array}}{\text{E bis.}}\end{array}\right\}$ comparanda $\left\{\begin{array}{l}\text{A quad.}\\ -\text{G plan.}\\ -\text{E quad.} \frac{1}{2}\end{array}\right.$

Et ſi proponatur denique $\begin{array}{l}\text{B ſolidum in A}\\ -\text{G plano in A quad bis}\\ -\text{A quad. quad.}\end{array}$ $\Big\{$ æqu. z pl. plano.

Inuerſis adfectionum notis Radix plana effingendi quadrati ſtaſtuetur $\left.\begin{array}{c}\dfrac{\begin{array}{c}\text{E in A}\\ +\text{B ſolido}\end{array}}{\text{E bis}}\end{array}\right\}$ comparanda $\left\{\begin{array}{l}\text{A quadrato}\\ +\text{G plano.}\\ +\text{E quad.} \frac{1}{2}\end{array}\right.$

Fit itaque omni caſu reductio quæ imperata eſt.

ALITER.

Æqualitatem Quadrato-quadrati affecti tam ſub Latere quam Quadrato, per medium Cubicæ Radicem habentis planam, ad Quadraticam reducere.

Proponatur $\left.\begin{array}{l}\text{A quad. quad.}\\ +\text{G plano in A quad.}\\ +\text{B ſolido in A}\end{array}\right\}$ æquari z plano-plano.

Oportet facere quod imperatum eſt.

Quoniam per ea quæ proponuntur facta Metatheſi,

A quad. quad. æquatur $\left\{\begin{array}{l}\text{Z plano-plano}\\ -\text{G plano in A quad.}\\ -\text{B ſolido in A.}\end{array}\right.$

Vtrobique addatur $\left\{\begin{array}{l}\text{E planum in A quad.}\\ +\text{E plano-plano} \frac{1}{4}\end{array}\right\}$ pars igitur parti rurſus adæquabitur, & ab illa quidem quum diuiditur ſubquadratice, oritur $\begin{array}{l}\text{A quadratum}\\ +\text{E plano} \frac{1}{2}.\end{array}$ diuiditor igitur quoque altera pars ſub quadratice, & idcirco effingatur quadratum abs commoda Radice, & illud comparetur, & adæquetur altera illi pars, vtpote ſtatuitor Radix.

$$\text{Lv.}\left\{\begin{array}{l}\dfrac{\text{B ſolidum}}{\text{E plan. 4.}}\\ -\text{G plan. 4.}\end{array}\right. -\text{Lv.}\left\{\begin{array}{l}\text{E plani}\\ -\text{G plano.}\end{array}\right\} \text{in A quadratum.}$$

Igitur B ſolido-ſolidum / E plano, 4. — G plano 4. —+ E plano, —G plano } in A quad. —B ſolido in A } æquabitur { Z plano-plano. —G plano in A quad. —B ſolido in A, —+ E plano in A quad. —+E plano-plano. $\frac{1}{4}$

Itaque—E plani cubus —G plano in E plani quad. —+ Z planoplano 4. in E plan. { æquabitur { B ſolido-ſolido —+Z plano-plan. in G. pl. 4.

E planum autem innoteſcat eſſe F planum.

Igitur B ſolidum / Lv. { F plani 4. —G plano 4. } —Lv. { F plani —G plano } in A q. } æquabitur { A quadrato —+F plano $\frac{1}{2}$.

Et ſi proponatur A quad. quad. —+G plano in A quad. —B ſolido in A. { æquari z plano-plano.

Radix plana effingendi quadrati ſtatuetur

Lv. { E plani —+G plano } in A q. —B ſolido / Lv. { E plani 4. —G plano 4 } comparanda } E plano $\frac{1}{2}$ —+A quadrato.

Et ſi proponatur denique—B ſolidum in A —G plano in A quad. —A quad. quad. { æqu. z plano-plano.

Radix plana effingendi quadrati ſtatuetur.

Lv. { E plani —G plano } in A q. —+B ſolido / Lv. { E plani 4. —G plano 4. } comparanda { E plano $\frac{1}{2}$ —+A quadrato.

Fit itaque omni caſu reductio quæ imperata eſt.

Ac ſecundum priorem quidem formulam, ordinata ſunt Theoremata hæc

THEOREMA I.

Secundum priorem formulam.

Si A quad. quad. —+G plano bis in A quad. —+B ſolido in A. } æquetur z plano-plano.

E quadrati cubus —+G plano 4 in E quadrati quadratum —+Z plano-plano 4 } —+G plano-plano 4 } in E quadratum. } æquetur B ſolido-ſolido.

Innoteſcat autem E eſſe D. A quad. + D in A } æquab. $\frac{\text{B ſolido}}{\text{D bis}}$ + D quadrato ½ — G plano.

1 QQ. + 6 Q. + 880 N. *æquatur* 1,800. *fit* 1 N 2.
1 C. + 12 Q. + 7,236 N. *æquatur* 774,400. *fit* 1 N. 64. *quadratum à Radice* 8.
1 Q. + 8 N. *æquabitur* 20. *fit* 1 N. 2.

THEOREMA II.

Si A quad. quad.
+ G plano bis in A quad.
— B ſolido in A.
} æquatur Z plano-plano.

E quadrati cubus
+ G plano-plano 4. in E quadrati quad.
+ Z plano-plano 4. }
+ G plano-plano 4. } in E quadratum
} æquatur B ſolido-ſolido.

Et E innoteſcat eſſe D. A quad. — D in A { æquabitur { $\frac{\text{B ſolido}}{\text{D 2}}$ — D quadrato — G plano.

1 QQ + 6 Q — 880 N. *æquatur* 1,800. *fit* 1 N. 10.
1 C + 12. Q + 7,236 N *æquatur* 774,400. *fit* 1 N. 64. *quadratum abs* 8.
1 Q — 8 N *æquatur* 20. *fit* 1 N. 10.

THEOREMA III.

Si A quad. quad.
— G plano bis in A quad.
+ B ſolido in A
} æquetur Z plano-plano.

E quadrati cubus
— G plano 4. in E quadrati quad.
+ Z plano-plano 4. }
+ G plano-plano 4. } in E quad.
} æquet. B ſolido-ſolido.

Et E innoteſcat eſſe D, A quad. + D in A. } æquabitur } $\frac{\text{B ſolido}}{\text{D 2}}$ — D quad. ½ + G plano.

1 QQ — 4 Q + 800 N. *æquatur* 1,600 *fit* 1 N. 2.
1 C. — 8 Q + 6,416 N. *æquatur* 640,000. *fit* 1 N 64. *quadratum abs* 8.
1 Q + 8 N. *æquatur* 20. *fit* 1 N. 2.

THEO-

THEOREMA. IIII.

Si A quad.quad.
— G plano bis in A quad
— B ſolido in A.
} æquetur z plano-plano.

E quadrati cubus
— G plano 4. in E quadrati quadrat.
+ Z plano-plano 4.
+ G plano-plano 4. } in E quad.
} æquet. B ſolido-ſolidum.

Et E innoteſcat eſſe D. A quad — D in A } æquabitur — B ſolido / D 2. — D quad. $\frac{1}{4}$ + G plano.

1 QQ — 4 Q — 800 N *æquatur* 1,600. *fit* 1 N. 10.
1 C — 8 Q + 6,416 N. *æquatur* 640.000. *fit* 1 N. 64. *quad. abs* 8.
1 Q — 8 N. *æquatur* 20. *fit* 1 N. 10.

THEOREMA V.

Si G planum bis in A quad.
+ B ſolido in A
— A quad.quad.
} æquetur z plano-plano.

E quadrati cubus
— G plano 4. in E quadrati quad
— Z plano-plano 4
+ G plano plano 4. } in E quad.
} æquet B ſolido-ſolidum.

Et E innoteſcat eſſe D, D in A — A quad. } æquabitur { D quad. $\frac{1}{4}$ — G plano — B ſolido / D 2.

44. Q + 720 N. — 1 QQ *æquatur* 1,600. *fit* 1 N 10. *vel* 2.
1 C. — 88 Q — 4,464 N. *æquatur* 518,400. *fit* 1 N. 144. *quadratum abs* 12.
12 N. — 1 Q *æquatur* 20. *fit* 1 N. 10 *vel* 2.

THEOREMA VI.

Si G planum bis in A
— B ſolido in A
— A quad. quad.
} æquetur z plano-plano.

E quadrati cubus.
— G plano 4 in E quadrati quad
+ Z plano-plano 4.
— G plano-plano 4 } in E quad
} æquet. B ſolido-ſolidum.

Et E innoteſcat eſſe D, D in A — A quad. } æquabitur { D quad. $\frac{1}{4}$ — G plano 4. — B ſolido / D 2.

O

114 Q. — 120 N. — 1 QQ. *æquatur* 200, *& fit* 1 N. 2. *vel* 10.
1 C — 228 Q. + 12,196 N. *æquatur* 14,400. *fit* 1 N. 144. *quad. abs* 12.
12 N. — 1 Q. *æquatur* 20. *fit* 1 N. 10. *vel* 2.

THEOREMA VII.

Si B ſolidum in A — G plano bis in A quad. — A quad. quad. } æquetur z plano-plano.

E quadrati cubus + G plano 4 in E quadrati quadratum — Z plano-plano 4. + G plano-plano 4. } in E quadratum. } æquetur B ſolido-ſolido.

Et E innoteſcat eſſe D, D in A - A quad. } æquabitur { D quad. $\frac{1}{2}$ — B ſolido / D 2. + G plano.

1,440 N. — 16 Q — 1 QQ. *æquatur* 2,800 *& fit* 1 N. 10. *vel* 2.
1 C. + 32 Q — 10,944 N. *æquatur* 2,073,600. *& fit* 1 N. 144. *quadratum abs* 12.
12 N. — 1 Q *æquatur* 20. *& fit* 1 N. 10 *vel* 2.

Ad poſteriorem autem formulam pertinent quæ ſequuntur.

THEOREMA I.

Secundum poſteriorem Formulam.

Si A quad. quad. + G plano in A quad. + B ſolido in A. } æquetur z plano-plano.

E plani cubus — G plano in E plani quad. + Z plano 4. in E planum. } æquetur B ſolido-ſolido + Z plano-plan. in G planum

Et innoteſcat E planum eſſe F planum.

+ Lv. { A quad. F plani - G plano } in A q } æquabitur Lv. { B ſolido / F plani 4. — G plano 4. — F plano $\frac{1}{2}$

1 QQ. + 6 Q. + 880 N. *æquatur* 1,800. *fit* 1 N. 2.
1 C — 6 Q. + 7,200 N. *æquatur* 817,600 *fit* 1 N. 70.
70 — 6 *eſt quadratum abs* 8.
1 Q. + 8 N. *æquabitur* 20 *fit* 1 N. 2.

THEOREMA. II.

Si A quad. quad. + G plano in A quad. — B ſolido in A } æquetur z plano-plano.

E plani cubus
— G plano in E plani quad. } æquetur { B ſolido-ſolido
→ Z plano plano 4 in E planum } { →Z plano-plano 4 in G planum.

Et innoteſcat E planum eſſe F planum.

A quadratum
-Lv { F plani / —G plano } in A q } æquab. $\frac{\text{B ſolido}}{\text{Lv.} \{ \text{F plani 4.} / \text{—G plano 4.} \}}$ —F plano ½.

1 Q Q → 6 Q — 800 N. *æquatur* 1, 800. *fit* 1 N. 10.

1 C. — 6 Q → 7,200 N. *æquatur* 817,600 *fit* 1 N. 70.

70 — 6 *eſt quadratum abs* 8.

1 Q — 8 N. *æquabitur* 10. *fit* 1 N. 10.

THEOREMA. III.

Si A quad. quad.
—G plano in A quad. } æqueturz plano-plano.
→B ſolido in A

E plani cubus.
→G plano in E plani quad. } æquetur { B ſolido-ſolido
→Z plano-plano 4 in B planum. } { —Z plano-plano 4. in G plan.

Et innoteſcat autem E planum eſſe F planum.

A quadratum
→Lv. { F plani / —G plano } in A q. æqu. $\frac{\text{B ſolido}}{\text{Lv.} \{ \text{F plani 4.} / \text{→G plano 4.} \}}$ — F plano ½.

1 Q Q — 4 Q → 800 N. *æquatur* 1,600. *fit* 1 N. 2.

1 C. → 4 Q → 6,400 N. *æquatur* 614, 400. *fit* 1 N. 60.

60 → 4 *faciunt quadratum abs* 8.

1 Q → 8 N. *æquatur* 20. *fit* 1 N. 2.

THEOREMA IIII.

Si A quad. quad.
—G plano in A quad. } æqueturz plano-plano.
—B ſolido in A

E plani cubus
→G plano in E plani quad. } æquetur { B ſolido-ſolido
→Z plano-plano 4 in E plan. } { →Z plano-plano 4 in G planum.

Et innoteſcat E planum eſſe F planum.

A quadratum
— Lv. { F plani / →G plano } in A q. } æquab. $\frac{\text{B ſolido}}{\text{Lv.} \{ \text{F plani. 4.} / \text{→G plano 4.} \}}$ —F plano ½.

1 Q Q. — 4 Q. — 800 N. *æquetur* 1, 600. *fit* 1 N. 10.

1 C. → 4 Q. → 64 N. *æquatur* 614,400. *fit* 1 N. 60.

60 → 4. *eſt quadratum abs* 8.

1 Q. — 8 N. *æquatur* 20. *fit* 1 N. 10.

THEOREMA V.

Si G planum in A quad.
+ B ſolido in A.
— A quad. quad. } æquetur Z plano-plano.

E plani cubus
+ G plano in E plani quad.
— Z plano-plano 4 in E planum. } æquetur { B ſolido-ſolido.
+ Z plano-plano 4 in G plan.

Et innoteſcat E planum eſſe F planum.

Lv. { F plani
+ G plano } in A qu.
— A quadrato } æquabitur F plano ½ — B ſolido / Lv. F plano 4. + G lano. 4.

44 Q. + 720 N. — 1 QQ. *æquatur* 1,600. *fit* 1 N. 10. *vel* 2.
1 C. + 44 Q. — 6,400 N. *æquatur* 800,000. *fit* 1 N. 100.
100 + 44. *eſt quadratum abs* 12.
12 N — 1 Q. *æquabitur* 20 *fit* 1 N. 10 *vel* 2.

THEOREMA VI.

Si G planum in A quad.
— B ſolido in A
— A quad. quad. } æquetur Z plano-plano.

E quadrati cubus
+ G plano in E plani quadratum
— Z plano-plano 4. in E planum } æquetur { B ſolido-ſolido.
+ Z plano-plano 4. in G plan.

Et E planum innoteſcat eſſe F planum.

Lv. { F plani
+ G plano } in A q.
— A quadrato. } æquabitur F plano ½ + B ſolido / Lv. { F plani 4. + G plano 4.

114 Q. — 120 N. — 1 QQ. *æquatur* 200. *fit* 1 N. 2. *vel* 10.
1 C. + 144 Q. — 800 N. *æquatur* 105,600. *fit* 1 N. 30.
30 + 114. *facit quadratum abs* 12.
12 N. — 1 Q. *æquatur* 20. *fit* 1 N. 2. *vel* 10.

THEOREMA VII.

Si B ſolido in A
— G plan in A quad.
— A quad. quad. } æquetur Z plano-plano.

E plani cubus
— G plano in E plani quad.
— Z plano-plano 4. in E planum } æquetur { B ſolido-ſolido
— Z plano-plano 4. in G plan.

Et E planum innoteſcat eſſe F planum.

Lv. { F plani / —G plano } in A q. / —A quadrato } æquabitur F plano $\frac{1}{2}$ —B ſolido / Lv. { F plani 4. / —G plano 4.

1,440 N. —16 Q. —1 QQ *æquatur* 2800. *fit* 1 N. 2 *vel* 10.

1 C. —16 Q —11,200 N. *æquatur* 1,894,400. *fit* 1 N. 160.

160 —16 *facit quadratum abs* 12.

12 N. —1 Q *æquatur* 20. *fit* 1 N. 2. *vel* 10.

Et quid attinet reliquas metatheſes perſequi, quum adfectiones ſub cubo euaneſcant expurgatione per vncias quadrantes. Hæc itaque ſunto ſatis ſuperque.

Quemadmodum Æquationes Cubicæ deprimuntur ad Quadraticas à Radice ſolida.

ſeu

De Duplicata Hypoſtaſi.

CAP. VII.

AEquè modus tranſmutandi qui dicitur Duplicata Hypoſtaſis, non minus elegans eſt & impendioſus, ad exhibendas Radicum Aſymmetrias in cubis aliquot ſub latere affectis, ac Zeteſin nouam inſtituendi cura, ex agnita ſingulari de qua initio præcedentis capitis monuimus, cuborum illorum conſtitutione.

Sequentia itaque iuuabit ſubiunxiſſe Problemata.

PROBLEMA I.

CVbum adfectum ſub Latere adfirmatè, ad Quadratum Radicem habens ſolidam, idemque adfectum, reducere.

Proponatur A cubus / +B plano ter in A } æquari Z ſolido bis.

Opprtet facere quod propoſitum eſt.

E quadratum / +A in E. } æquetur B plano.

Vnde B planum ex huiuſmodi æquationis conſtitutione, intelligitur rectangulum ſub duobus Lateribus quorum minus eſt E,

differentia à maiore A. igitur (B planum — E quad) / E erit A.

Quare
B plano-plano-planum.
— E quad in B plano-planum ter
+ E quad. quad. in B planum ter
— E cubo-cubo
/ E cubo.
+ B plano-plano ter
— B plano in E quad. ter
/ E
} æquabitur z ſolido bis.

Et omnibus per E cubum ductis & ex arte concinnatis.

E cubi quadratum
+ Z ſolido bis in E cubum } æquabitur B plani cubo.

Quæ æquatio eſt quadrati affirmate affecti, Radicem habentis ſolidam ;facta itaque reductio eſt quæ imperabatur.

Conſectarium.

ITaque ſi. A cubus + B plano ter } æquetur z ſolido bis.

L { B plano-plano-plani + Z ſolido ſolido. } æquetur D cubo.
— Z ſolido.

(B planum — D quadrat.) / D } fit A de qua quæritur.

Si 1 C. + 81 N. *æquetur* 702.
quoniam l 19,683. + 123,201 *ſeu* 142,884, *ſeu denique* 378.
multatus ſolido 351. *eſt* 27 *cubus à Latere* 3.
Ideo 27 *ſeu* 6 *eſt* 1 N. *de qua quæritur.*
— 9
/ 3

Aliter & ſecundo.

E quadratum — A in E. } æquetur B plano.

Vnde B planum ex huiuſmodi æquationis conſtitutione, intelligitur rectangulum ſub duobus lateribus, quorum maius eſt E, exceſſus vero eiuſdem ſupra minorem A. igitur

(E quadratum — B plano) / E } æquabitur A.

Quare per ea quæ proponuntur, omnibus ex arte concinnatis.

E cubi quadratum
— Z ſolido bis in E cubum } æquabitur B plani cubo.

Quæ æquatio eſt quadrati negate adfecti, Radicem habentis ſo-

lidam. Facta itaque eſt rurſus reductio quæ imperabatur.

Conſectarium ſecundum.

ITaque ſi A cubus + B plano ter in A, æquetur Z ſolido bis.

L { B plano-plano-plani + Z ſolido ſolido. } + Z ſolido, æquetur G cubo.

$\frac{\text{G quadratum} - \text{B plano}}{\text{G}}$ æquabitur A.

Si 1 C + 81 N. *æquetur* 702.
quoniam 378. *plus* 351 *eſt* 729. *Cubus à Latere* 9. *ideo* 81 *ſeu* 6. *eſt* 1 N. *de qua quæritur.* $\frac{-27}{9}$

Conſectarium.

E duobus antedictis conſectariis.

Denique ſunt duo latera vnum idemque minus D, alterum idemque maius G, quorum differentia eſt A de qua quæritur.

Itaque L v c. L { B plano-plano plani + Z ſolido-ſolido. } + Z ſolido.

— L v c. L { B plano-plano-plani + Z ſolido-ſolido } — Z ſolido.

Eſt A quæſita.

Si 1 C + 6 N. *æquetur* 2.
L c 4 — L c 2. *eſt* 1 N *de qua quæritur.*

PROBLEMA II.

CVbum adfectum ſub Latere negatè, ad planum ſub Radice ſolida negatum de Quadrato, reducere.

Oportet autem in æquatione propoſitâ, cubum e triente coefficientis adfectionis, cedere ſolidi comparationis ſub quadruplo quadrato.

Proponatur A cubus — B plano ter in A æquari Z ſolido bis.

Oportet facere quod propoſitum eſt.

A in E — E quadrato æquetur B plano.

Vnde B planum ex huiuſmodi æquationis conſtitutione, intelligitur rectangulum ſub duobus lateribus, quorum maius minuſ-

ue eſt E, ſumma vero minoris ac maioris A.

Igitur $\dfrac{\text{B planum} + \text{B quadrato}}{\text{E}}$ æquabitur A.

Quare $\left.\begin{array}{l} \dfrac{\begin{array}{l}\text{B plano-plano-planum.} \\ + \text{E quad. in B plano-planum ter} \\ + \text{E quad. quad. in B planum ter} \\ + \text{E cubo-cubo.}\end{array}}{\text{E cubo.}} \\ \dfrac{- \text{B plano-plano ter} \\ - \text{B plano in E quad. ter}}{\text{E}} \end{array}\right\}$ æquabitur z ſolido bis.

Et omnibus per E cubum ductis & ex arte concinnatis.

$\begin{array}{l}\text{Z ſolidum bis in E cubum} \\ - \text{E cubi quadrato.}\end{array}$ æquabitur B plani cubo.

Quæ æquatio eſt plani ſub Radice ſolida, negati de quadrato. Facta itaque eſt Reductio quæ imperabatur.

Apparet autem ex æquationis illius ad quam reductio facta eſt proprietate, z ſolidi quadratum præſtare debere B plani cubo: quo pertinet appoſita lex problemati.

Conſectarium.

Itaque ſi $\begin{array}{l}\text{A cubus} \\ - \text{B plano ter in A}\end{array}$ æquetur z ſolido bis.

Lvc. Z ſolidi + L $\left\{\begin{array}{l}\text{Z ſolido-ſolidi} \\ - \text{B plano plano-plano.}\end{array}\right.$

+ Lvc. Z ſolidi − L $\left\{\begin{array}{l}\text{Z ſolido-ſolidi} \\ - \text{B plano-plano-plano.}\end{array}\right.$

Eſt A de qua quæritur.

Si 1 C. — 81 N. æquetur 756.

Quoniam 378 + 351. eſt 729, cubus à latere 9.

Ideo 9 + 3 id eſt 12. eſt 1 N. de quâ quæritur.

De Canonica Æquationum Tranſmutatione, vt coefficientes ſub-graduales ſint quæ præſcribuntur.

CAP. VIII.

Ad impendia quoque logiſtica confert æquationes ita præparare, vt coefficientes æquationum vel comparationum homogenea ſint quæ præſcribuntur, quod libere licet ex Canoni-

ca

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eâ tranſmutandi doctrinâ. Statuitor coefficiens vnitas, vſus igitur impendij vel ex eo liquet, quod poteſtates adfectæ ſub vnitate & gradu quocunque, (ſi modo Radix eſt numerus.) non aliter reſoluuntur ac ſi eſſent puræ. neque enim negotium conturbat habendâ alioqui (ſed quæ ex iſthoc opere, ſaluâ ſit.) Parabolarum quarum quæque vnitas eſt, ratio.

Et ſi Radix inuenta non eſt numerus, ipſam Radicem de qua quæritur non eſſe numerum ſtatim conuincitur.

I C + I N. æquatur 10. quoniam proximus cubus eſt 8. cuius Radix eſt 2, quæ ducta in vnitatem & adiuncta 8. facit 10. ideo Radix quæſita eſt 2.

Aequè I C — I N æquetur 24. quoniam proxime minor cubus eſt 8, cuius Radix eſt 2. & adſcita vnitate 3. ſub qua & vnitate facto ſolido, quum multatur cubus ex 3. relinquitur 24. ideo Radix quæſita eſt 3.

Proponatur autem I C. + I N. æquari 9. quoniam proximus cubus eſt 8, cuius Radix eſt 2. qua ducta in vnitatem & adiuncta 8, facit 10. non etiam 9. ideo I N eſt Radix irrationalis.

Æque I C — I N. æquetur 25. quoniam proxime minor cubus eſt 8. cuius Radix eſt 2. & adſcita vnitate 3. ſub quâ & vnitate facto ſolido quum multatur cubus ex 3. relinquitur 24, non etiam 25. ideo I N eſt irrationalis.

Oportet autem ad huiuſmodi inſtituendæ tranſmutationis opus, coefficientem quæ imperatur coefficienti æquationis propoſitæ eſſe congenerem, & ſi quidem Radix æquationis propoſitæ eſt eiuſdem quoque generis, concedetur eſſe vt coefficiens propoſitæ æquationis, ad coefficientem imperatam, ita Radix de qua quærebatur, ad nouam ſtatuendam Radicem.

Sin coefficiens gradui Radicis genere communicet, concedetur eſſe vt coefficiens propoſitæ æquationis, ad coefficientem imperatam, ita gradus æque-altus Radicis de qua quærebatur, ad gradum æque altum nouæ ſtatuendæ Radicis.

Et per reſolutionem conceſſi Analogiſmi, exhibebitur ſub nouâ ſpecie valor Radicis quæſitæ, & ſua propoſita æqualitas dirigetur, & ordinabitur noua.

Quod, vt vno aut altero exemplo fiat apertius.

Et ſit E planum ſeu D planum differentia ſubquadrupla inter quadratum aggregati extremarum, adiuncti triplo aggregati mediarum, & quadratum aggregati omnium.

Vnde Lv. { B quad 4. + D plano 4. } eſt aggregatum extremarum, plus triplo aggregati mediarum, & ſit A compoſita ex tribus primis ſiue ex tribus poſtremis.

Sint proportionales continuè 1. 2. 4. 8.

15 C. — 1 QQ. *æquatur* 2,744.

Igitur 21 N. — 1 Q. *æquabitur* 98 *& ſit* 1 N. 7. *vel*. 14.

Nota eſt autem 21. *quoniam* 1 C. — 10,976 N. *æquatur* 617,400.

Et ſit 1 N 126. *differentia ſubquadrupla inter* 225. *&* 729.

Vnde dignoſcitur l. 729. *id eſt longitudo* 27. *quæ adiecta longitudini* 15. *facit* 42. *duplum ipſius* 21.

PROBLEMA. III.

AEqualitatem Quadrato-quadrati ádfecti tam ſub Latere quam Quadrato, per medium Cubicæ Radicem habentis planam, ad Quadraticam deprimere.

Proponatur { A quadrato-quadratum. + G plano in A quad. bis. + B ſolido in A. } æquari Z plano-plano.

Oportet facere quod imperatum eſt.

Sane ſi quadratum effingatur abs { A quad. + G plano. + E quad. $\frac{1}{2}$ } erit illud { A quad. quad. + G plan. plano. + G quad in A qu. bis. + E quad. quad. $\frac{1}{4}$ + E quad. in A quad. + G plan. in E quad. }

Quoniam igitur ex ijs quæ propoſita ſunt, adhibita Metatheſi. { A quadrato-quadratum + G plano in A qudratum bis } æquatur { Z plano-plano — B ſolido in A. }

Vtrique igitur æquationis parti addatur id quod deficit ab effecto à ſtatutâ Radice plana quadrato, ergo hac æqualium, æqualibus additione, rurſum pars parti æqualis erit.

Iam vtraq; pars diuiditor ſub quadratice, illic reuocatâ ad Analyſin Geneſi, orietur manifeſto { A quadratum. + G plano + E quad. $\frac{1}{2}$ }

Quod ſi altera quoque æqualitatis pars poſſet diuidi ſub quadraticè, quod orîretur, foret Radicibus illis planis ex prima parte ortiuis æquale.

Effingendum eſt igitur quadratum à Radice planâ, & illud alteri æqualitatis parti, id eſt z plano-plano vna cum ſuis adfectionibus comparandum & ad æquandum, vt Radices quoque comparentur & adæquentur inter ſe; & ſtatuatur idcirco Radix illa plana effingendi quadrati $\frac{\text{B ſolidum}}{\text{E bis.}}$ — E in A, ſic enim in comparatione euaneſcent adfectiones ſub A & gradibus, & incidetur in æqualitatem de E, quo tendendum eſt.

Effictum igitur quadratum erit.

+ B ſolido-ſolidum / E quad. 4.
+ E quad. in A quad.
— B ſolido in A.
æquale
Z plano-plano.
— B ſolid. in A
+ G plan. plano
+ E quad. quad. $\frac{1}{4}$
+ E quad. in A quad.
+ G plan in E quad.

Et deletis vtrinque adfectionibus ſub A & A quadrato.

B ſolido-ſolidum / Eq. 4.
æquabitur
Z plano-plano
+ G plano-plano
+ E quad. quad. $\frac{1}{4}$
+ G plan in E quad.

Et omnibus in E quadratum 4. ductis & rite ordinatis.

E quadrati cubus
+ G plano 4 in E quadrati quad.
+ Z plano-plano 4. } in E quad.
+ G plano-plano 4 }
æquabitur B ſolido-ſolido.

Innoteſcat autem E eſſe D. Aq. + D in A æquabitur
B ſolido / D 2
— G plano
+ D quadrato $\frac{1}{4}$

Et ſi proponatur
A quad. quad.
+ G plano in A quadrat. bis
— B ſolido in A
æquari z plano-plano.

Radix effingendi quadrati ſtatuetur.

B ſolidum / E bis
— E in A
comparanda
A quadrato
+ E plano
+ E quad. $\frac{1}{2}$

Et ſi proponantur
A quad. quad.
— G plano A quad. bis.
— B ſolido in A.
æquari z plano-plano.

Radix plana effingendi quadrati ſtatuetur.

B ſolidum / E bis.
+ E in A
comparanda.
A quadrato
— G plano
+ E quad. $\frac{1}{2}$

Et ſi proponatur
G plan. in A quad. bis.
+ B ſolido in A
— A quad. quad.
æquari z plano-plano.

Inuerſis adfectionum notis, Radix plana effingendi quadrati

ſtatuetur quoque $\frac{\text{B ſolidum}}{\text{E bis}}$ + E in A } comparanda { A quadrato — G plano + E quadrato. $\frac{1}{2}$

Et ſi proponatur G plan. in A quad. bis. — B ſolido in A — A quad quadrato. { æquari z plano-plano.

Inuerſis adfectionum notis, Radix plana effingendi quadrati ſtatuetur E in A $+\frac{\text{B ſolido}}{\text{E bis.}}$ } comparanda { A quad. — G plan. — E quad. $\frac{1}{2}$

Et ſi proponatur denique B ſolidum in A — G plano in A quad bis — A quad. quad. { æqu. z pl. plano.

Inuerſis adfectionum notis Radix plana effingendi quadrati ſtatuetur E in A $+\frac{\text{B ſolido}}{\text{E bis}}$ } comparanda { A quadrato + G plano. + E quad. $\frac{1}{2}$

Fit itaque omni caſu reductio quæ imperata eſt.

ALITER.

AEqualitatem Quadrato-quadrati affecti tam ſub Latere quam Quadrato, per medium Cubicæ Radicem habentis planam, ad Quadraticam reducere.

Proponatur A quad. quad. + G plano in A quad. + B ſolido in A } æquari z plano-plano.

Oportet facere quod imperatum eſt.

Quoniam per ea quæ proponuntur facta Metatheſi,

A quad. quad. æquatur { Z plano-plano — G plano in A quad. — B ſolido in A.

Vtrobique addatur { E planum in A quad. + E plano-plano $\frac{1}{4}$ } pars igitur parti rurſus adæquabitur, & ab illa quidem quum diuiditur ſubquadratice, oritur A quadratum + E plano $\frac{1}{2}$. diuiditor igitur quoque altera pars ſub quadratice, & idcirco effingatur quadratum abs commoda Radice, & illud comparetur, & adæquetur altera illi pars, vtpote ſtatuitor Radix.

Lv. { $\frac{\text{B ſolidum}}{\text{E plan. 4.}}$ — G plan. 4. — Lv. { E plani — G plano. } in A quadratum.

Igitur $\frac{\text{B ſolido-ſolidum}}{\text{E plano. 4. — G plano 4.}}$ + E plano — G plano } in A quad. — B ſolido in A } æquabitur { Z plano-plano. — G plano in A quad. — B ſolido in A, + E plano in A quad. + E plano-plano. $\frac{1}{4}$

Itaque E plani cubus — G plano in E plani quad. + Z planoplano 4. in E plan. } æquabitur { B ſolido-ſolido + Z plano-plan. in G. pl. 4.

E planum autem innoteſcat eſſe F planum.

Igitur $\frac{\text{B ſolidum}}{\text{Lv. \{F plani 4. — G plano 4.}}$ — Lv. { F plani — G plano } in A q. } æquabitur { A quadrato + F plano $\frac{1}{2}$

Et ſi proponatur A quad.quad. + G plano in A quad. — B ſolido in A. } æquari z plano-plano.

Radix plana effingendi quadrati ſtatuetur

Lv. { E plani + G plano } in A q — $\frac{\text{B ſolido}}{\text{Lv. \{E plani 4. — G plano 4}}$ } comparanda } E plano $\frac{1}{2}$ + A quadrato.

Et ſi proponatur denique B ſolidum in A — G plano in A quad. — A quad. quad. } æqu. z plano-plano.

Radix plana effingendi quadrati ſtatuetur.

Lv. { E plani — G plano } in A q. + $\frac{\text{B ſolido}}{\text{Lv. \{E plani 4. — G plano 4.}}$ } comparanda { E plano $\frac{1}{2}$ — + A quadrato.

Fit itaque omni caſu reductio quæ imperata eſt.

Ac ſecundum priorem quidem formulam, ordinata ſunt Theoremata hæc

THEOREMA I.

Secundum priorem formulam.

Si A quad.quad. + G plano bis in A quad. + B ſolido in A. } æquetur z plano-plano.

E quadrati cubus + G plano 4 in E quadrati quadratum + Z plano-plano 4 + G plano-plano 4 } in E quadratum. } æquetur B ſolido-ſolido.

Innoteſcat autem E eſſe D. $\left.\begin{matrix} \text{A quad.} \\ +\text{D in A} \end{matrix}\right\}$ æquab. $\dfrac{\text{B ſolido}}{\text{D bis}}$ $+$ D quadrato $\frac{1}{4}$ $-$ G plano.

1 QQ. + 6 Q. + 880 N. *æquatur* 1, 800. *fit* 1 N 2.
1 C. + 12 Q. + 7, 236 N. *æquatur* 774, 400. *fit* 1 N. 64. *quadratum à Radice* 8.
1 Q. + 8 N. *æquabitur* 20. *fit* 1 N. 2.

THEOREMA II.

Si $\left.\begin{matrix} \text{A quad. quad.} \\ +\text{G plano bis in A quad.} \\ -\text{B ſolido in A.} \end{matrix}\right\}$ æquatur z plano-plano.

$\left.\begin{matrix} \text{E quadrati cubus} \\ +\text{G plano-plano 4. in E quadrati quad.} \\ \left.\begin{matrix} +\text{Z plano-plano 4.} \\ +\text{G plano-plano 4.} \end{matrix}\right\} \text{in E quadratum} \end{matrix}\right\}$ æquatur B ſolido-ſolido.

Et E innoteſcat eſſe D. $\left.\begin{matrix} \text{A quad.} \\ -\text{D in A} \end{matrix}\right\}$ æquabitur $\left\{\begin{matrix} \dfrac{\text{B ſolido}}{\text{D 2}} \\ -\text{D quadrato} \\ -\text{G plano.} \end{matrix}\right.$

1 QQ + 6 Q — 880 N. *æquatur* 1, 800. *fit* 1 N. 10.
1 C + 12. Q + 7, 236 N *æquatur* 774, 400. *fit* 1 N. 64. *quadratum abs* 8.
1 Q — 8 N *æquatur* 20. *fit* 1 N. 10.

THEOREMA III.

Si $\left.\begin{matrix} \text{A quad. quad.} \\ -\text{G plano bis in A quad.} \\ +\text{B ſolido in A} \end{matrix}\right\}$ æquetur z plano-plano.

$\left.\begin{matrix} \text{E quadrati cubus} \\ -\text{G plano 4. in E quadrati quad.} \\ \left.\begin{matrix} +\text{Z plano-plano 4.} \\ +\text{G plano-plano 4.} \end{matrix}\right\} \text{in E quad.} \end{matrix}\right\}$ æquet. B ſolido-ſolido.

Et E innoteſcat eſſe D, $\left.\begin{matrix} \text{A quad.} \\ +\text{D in A.} \end{matrix}\right\}$ æquabitur $\left\{\begin{matrix} \dfrac{\text{B ſolido}}{\text{D 2}} \\ -\text{D quad.} \frac{1}{4} \\ +\text{G plano.} \end{matrix}\right.$

1 QQ — 4 Q + 800 N. *æquatur* 1, 600 *fit* 1 N. 2.
1 C. — 8 Q + 6, 416 N. *æquatur* 640, 000. *fit* 1 N 64. *quadratum abs* 8.
1 Q + 8 N. *æquatur* 20. *fit* 1 N. 2.

THEO-

THEOREMA. IIII.

Si A quad. quad.
— G plano bis in A quad
— B ſolido in A.
} æquetur z plano-plano.

E quadrati cubus
— G plano 4. in E quadrati quadrat.
+ Z plano-plano 4.
+ G plano-plano 4. } in E quad.
} æquet. B ſolido-ſolidum.

Et E innoteſcat eſſe D. A quad. — D in A } æquabitur — B ſolido / D 2. — D quad. ½ + G plano.

1 QQ — 4 Q + 800 N æquatur 1,600. fit 1 N. 10.
1 C — 8 Q + 6,416 N. æquatur 640,000. fit 1 N. 64. quad. abs 8.
1 Q — 8 N. æquatur 20. fit 1 N. 10.

THEOREMA V.

Si G planum bis in A quad.
+ B ſolido in A
— A quad. quad.
} æquetur z plano-plano.

E quadrati cubus
— G plano 4. in E quadrati quad
— Z plano-plano 4
+ G plano plano 4. } in E quad.
} æquet B ſolido-ſolidum.

Et E innoteſcat eſſe D, D in A — A quad. } æquabitur { D quad. ½ — G plano — B ſolido / D 2.

44. Q + 720 N. — 1 QQ. æquatur 1,600. fit 1 N 10. vel 2.
1 C. — 88 Q — 4,464 N. æquatur 518,400. fit 1 N. 144. quadratum abs 12.
12 N. — 1 Q æquatur 20. fit 1 N. 10 vel 2.

THEOREMA VI.

Si G planum bis in A
— B ſolido in A
— A quad. quad.
} æquetur z plano-plano.

E quadrati cubus.
— G plano 4 in E quadrati quad
+ Z plano-plano 4.
— G plano-plano 4 } in E quad
} æquet. B ſolido-ſolidum.

Et E innoteſcat eſſe D, D in A — A quad. } æquabitur { D quad. ½ — G plano 4. — B ſolido / D 2.

114 Q. — 120 N. — 1 QQ. *æquatur* 200, *& fit* 1 N. 2. *vel* 10.
1 C — 228 Q. + 12, 196 N. *æquatur* 14, 400. *fit* 1 N. 144. *quad. abs* 12.
12 N. — 1 Q. *æquatur* 20. *fit* 1 N. 10. *vel* 2.

THEOREMA VII.

Si B ſolidum in A — G plano bis in A quad. — A quad. quad. } æquetur z plano-plano.

E quadrati cubus + G plano 4 in E quadrati quadratum — Z plano-plano 4. + G plano-plano 4. } in E quadratum. } æquetur B ſolido-ſolido.

Et E innoteſcat eſſe D, D in A - A quad. } æquabitur { D quad. + G plano. } $\frac{1}{2}$ — $\frac{\text{B ſolido}}{\text{D } 2.}$

1,440 N. — 16 Q. — 1 QQ. *æquatur* 2,800 *& fit* 1 N. 10. *vel* 2.
1 C. + 32 Q. — 10, 944 N. *æquatur* 2, 073, 600. *& fit* 1 N. 144. *quadratum abs* 12.
12 N. — 1 Q. *æquatur* 20. *& fit* 1 N. 10 *vel* 2.

Ad poſteriorem autem formulam pertinent quæ ſequuntur.

THEOREMA I.

Secundum poſteriorem Formulam.

Si A quad. quad. + G plano in A quad. + B ſolido in A. } æquetur z plano-plano.

E plani cubus — G plano in E plani quad. + Z plano 4. in E planum. } æquetur B ſolido-ſolido + Z plano-plan. in G planum

Et innoteſcat E planum eſſe F planum.

A quad. + Lv. { F plani - G plano } in A q } æquabitur $\frac{\text{B ſolido}}{\text{Lv. \{ F plani 4. — G plano 4.}}$ — F plano $\frac{1}{2}$.

1 QQ. + 6 Q. + 880 N. *æquatur* 1, 800. *fit* 1 N. 2.
1 C — 6 Q. + 7, 200 N. *æquatur* 817, 600 *fit* 1 N. 70.
70 — 6 *eſt quadratum abs* 8.
1 Q. + 8 N. *æquabitur* 20 *fit* 1 N. 2.

THEOREMA. II.

Si A quad. quad. + G plano in A quad. — B ſolido in A } æquetur z plano-plano.

$$\left.\begin{array}{l}\text{E plani cubus}\\ -\text{G plano in E plani quad.}\\ +\text{Z plano-plano 4 in E planum}\end{array}\right\}\text{æquetur}\left\{\begin{array}{l}\text{B ſolido-ſolido}\\ +\text{Z plano-plano 4 in G planum.}\end{array}\right.$$

Et innoteſcat E planum eſſe F planum.

$$-\text{Lv}\left\{\begin{array}{l}\text{A quadratum}\\ \left.\begin{array}{l}\text{F plani}\\ -\text{G plano}\end{array}\right\}\text{in A q}\end{array}\right\}\text{æquab.}\ \frac{\text{B ſolido}}{\text{Lv.}\left\{\begin{array}{l}\text{E plani 4.}\\ -\text{G plano 4.}\end{array}\right.}-\text{F plano}\ \tfrac{1}{2}.$$

1 QQ. + 6 Q. — 800 N. *æquatur* 1, 800. *fit* 1 N. 10.

1 C. — 6 Q. + 7,200 N. *æquatur* 817, 600. *fit* 1 N. 70.

70 — 6 *eſt quadratum abs* 8.

1 Q — 8 N. *æquabitur* 20. *fit* 1 N. 10.

THEOREMA. III.

$$\text{Si}\left.\begin{array}{l}\text{A quad. quad.}\\ -\text{G plano in A quad.}\\ +\text{B ſolido in A}\end{array}\right\}\text{æquetur Z plano-plano.}$$

$$\left.\begin{array}{l}\text{E plani cubus.}\\ +\text{G plano in E plani quad.}\\ +\text{Z plano-plano 4 in B planum.}\end{array}\right\}\text{æquetur}\left\{\begin{array}{l}\text{B ſolido-ſolido}\\ -\text{Z plano-plano 4 in G plan.}\end{array}\right.$$

Et innoteſcat autem E planum eſſe F planum.

$$+\text{Lv.}\left\{\begin{array}{l}\text{A quadratum}\\ \left.\begin{array}{l}\text{F plani}\\ -\text{G plano}\end{array}\right\}\text{in A q.}\end{array}\right.\ \text{æqu.}\left\{\frac{\text{B ſolido}}{\text{Lv.}\left\{\begin{array}{l}\text{F plani 4.}\\ +\text{G plano 4.}\end{array}\right.}\right.-\text{F plano}\ \tfrac{1}{2}.$$

1 QQ. — 4 Q. + 800 N. *æquatur* 1,600. *fit* 1 N. 2.

1 C. + 4 Q. + 6,400 N. *æquatur* 614, 400. *fit* 1 N. 60.

60 + 4 *faciunt quadratum abs* 8.

1 Q + 8 N. *æquatur* 20. *fit* 1 N. 2.

THEOREMA IIII.

$$\text{Si}\left.\begin{array}{l}\text{A quad. quad.}\\ -\text{G plano in A quad.}\\ -\text{B ſolido in A}\end{array}\right\}\text{æquetur Z plano-plano.}$$

$$\left.\begin{array}{l}\text{E plani cubus}\\ +\text{G plano in E plani quad.}\\ +\text{Z plano-plano 4. in E plan.}\end{array}\right\}\text{æquetur}\left\{\begin{array}{l}\text{B ſolido-ſolido}\\ +\text{Z plano-plano 4 in G planum.}\end{array}\right.$$

Et innoteſcat E planum eſſe F planum.

$$-\text{Lv.}\left\{\begin{array}{l}\text{A quadratum}\\ \left.\begin{array}{l}\text{F plani}\\ +\text{G plano}\end{array}\right\}\text{in A q.}\end{array}\right\}\text{æquab.}\ \frac{\text{B ſolido}}{\text{Lv.}\left\{\begin{array}{l}\text{F plani. 4.}\\ +\text{G plano 4.}\end{array}\right.}-\text{F plano}\ \tfrac{1}{2}.$$

1 QQ. — 4 Q. — 800 N. *æquetur* 1, 600. *fit* 1 N. 10.

1 C. + 4 Q. + 64 N. *æquatur* 614,400. *fit* 1 N. 60.

60 + 4. *eſt quadratum abs* 8.

1 Q. — 8 N. *æquatur* 20. *fit* 1 N. 10.

THEOREMA V.

Si G planum in A quad.
+ B ſolido in A.
— A quad. quad.
} æquetur z plano-plano.

E plani cubus
+ G plano in E plani quad.
— Z plano-plano 4 in E planum.
} æquetur { B ſolido-ſolido.
+ Z plano-plano 4 in G plan.

Et innoteſcat E planum eſſe F planum.

Lv. { F plani
+ G plano } in A qu.
— A quadrato
} æquabitur F plano $\frac{1}{2}$ — B ſolido / Lv. F plano 4. + Glano. 4.

44 Q. + 720 N. — 1 QQ. *æquatur* 1,600. *fit* 1 N. 10. *vel* 2.
1 C. + 44 Q. — 6, 400 N. *æquatur* 800,000. *fit* 1 N. 100.
100 + 44. *eſt quadratum abs* 12.
12 N — 1 Q. *æquabitur* 20 *fit* 1 N. 10 *vel* 2.

THEOREMA VI.

Si G planum in A quad.
— B ſolido in A
— A quad.quad.
} æquetur z plano-plano.

E quadrati cubus
+ G plano in E plani quadratum
— Z plano-plano 4. in E planum
} æquetur { B ſolido-ſolido.
+ Z plano-plano 4. in G plan.

Et E planum innoteſcat eſſe F planum.

Lv. { F plani
+ G plano } in A q.
— A quadrato.
} æquabitur F plano $\frac{1}{2}$ + B ſolido / Lv. { F plani 4. + G plano 4.

114 Q. — 120 N. — 1 QQ. *æquatur* 200. *fit* 1 N. 2. *vel* 10.
1 C. + 144 Q — 800 N. *æquatur* 105,600. *fit* 1 N. 30.
30 + 114. *facit quadratum abs* 12.
12 N. — 1 Q *æquatur* 20. *fit* 1 N. 2. *vel* 10.

THEOREMA VII.

Si B ſolido in A
— G plan in A quad.
— A quad.quad.
} æquetur z plano-plano.

E plani cubus
— G plano in E plani quad.
— Z plano-plano 4. in E planum
} æquetur { B ſolido-ſolido
— Z plano-plano 4. in G plan.

Et E planum innoteſcat eſſe F planum.

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Lv. { F plani / — G plano } in A q. / — A quadrato } æquabitur F plano $\frac{1}{2}$ — B ſolido / Lv. { F plani 4. / — G plano 4.

1,440 N. — 16 Q. — 1 QQ *æquatur* 2800. *fit* 1 N. 2 *vel* 10.

1 C. — 16 Q — 11,200 N. *æquatur* 1,894,400. *fit* 1 N. 160.

160 — 16 *facit quadratum abs* 12.

12 N. — 1 Q *æquatur* 20. *fit* 1 N. 2. *vel* 10.

Et quid attinet reliquas metatheſes perſequi, quum adfectiones ſub cubo euaneſcant expurgatione per vncias quadrantes. Hæc itaque ſunto ſatis ſuperque.

Quemadmodum Æquationes Cubicæ deprimuntur ad Quadraticas à Radice ſolida.

ſeu

De Duplicata Hypoſtaſi.

CAP. VII.

Aequè modus tranſmutandi qui dicitur Duplicata Hypoſtaſis, non minus elegans eſt & impendioſus, ad exhibendas Radicum Aſymmetrias in cubis aliquot ſub latere affectis, ac Zeteſin nouam inſtituendi cura, ex agnita ſingulari de qua initio præcedentis capitis monuimus, cuborum illorum conſtitutione.

Sequentia itaque iuuabit ſubiunxiſſe Problemata.

PROBLEMA I.

CVbum adfectum ſub Latere adfirmatè, ad Quadratum Radicem habens ſolidam, idemque adfectum, reducere.

Proponatur A cubus + B plano ter in A } æquari z ſolido bis.

Opprtet facere quod propoſitum eſt.

E quadratum + A in E. } æquetur B plano.

Vnde B planum ex huiuſmodi æquationis conſtitutione, intelligitur rectangulum ſub duobus Lateribus quorum minus eſt E,

differentia à maiore A. igitur $\frac{\text{B planum} - \text{E quad}}{\text{E}}$ erit A.

Quare
$$\left.\begin{array}{l} \frac{\text{B plano-plano-planum} - \text{E quad in B plano-planum ter} + \text{E quad.quad.in B planum ter} - \text{E cubo-cubo}}{\text{E cubo.}} \\ + \text{B plano-plano ter} \\ \frac{- \text{B plano in E quad.ter}}{\text{E}} \end{array}\right\} \text{æquabitur z ſolido bis.}$$

Et omnibus per E cubum ductis & ex arte concinnatis.
$$\left.\begin{array}{l} \text{E cubi quadratum} \\ + \text{Z ſolido bis in E cubum} \end{array}\right\} \text{æquabitur B plani cubo.}$$

Quæ æquatio eſt quadrati affirmate affecti, Radicem habentis ſolidam; facta itaque reductio eſt quæ imperabatur.

Conſectarium.

Itaque ſi. $\left.\begin{array}{l} \text{A cubus} \\ + \text{B plano ter} \end{array}\right\}$ æquetur z ſolido bis.

$$\text{L}\left\{\begin{array}{l} \text{B plano-plano-plani} \\ + \text{Z ſolido ſolido.} \end{array}\right. \left.\vphantom{\begin{array}{l} a \\ b \end{array}}\right\} \text{æquetur D cubo.}$$
$$- \text{Z ſolido.}$$

$$\left.\frac{\text{B planum} - \text{D quadrato}}{\text{D}}\right\} \text{fit A de qua quæritur.}$$

Si 1 C. + 81 N. *æquetur* 702.

quoniam l 19,683. + 123,201 *ſeu* 142,884, *ſeu denique* 378.
multatus ſolido 351. *eſt* 27 *cubus à Latere* 3.

Ideo $\frac{27 - 9}{3}$ *ſeu* 6 *eſt* 1 N. *de qua quæritur.*

Aliter & ſecundo.

$$\left.\begin{array}{l} \text{E quadratum} \\ - \text{A in E.} \end{array}\right\} \text{æquetur B plano.}$$

Vnde B planum ex huiuſmodi æquationis conſtitutione, intelligitur rectangulum ſub duobus lateribus, quorum maius eſt E, exceſſus vero eiuſdem ſupra minorem A. igitur

$$\left.\frac{\text{E quadratum} - \text{B plano}}{\text{E}}\right\} \text{æquabitur A.}$$

Quare per ea quæ proponuntur, omnibus ex arte concinnatis.
$$\left.\begin{array}{l} \text{E cubi quadratum} \\ - \text{Z ſolido bis in E cubum} \end{array}\right\} \text{æquabitur B plani cubo.}$$

Quæ æquatio eſt quadrati negate adfecti, Radicem habentis ſo-

lidam. Facta itaque eſt rurſus reductio quæ imperabatur.

Conſectarium ſecundum.

ITaque ſi $\left.\begin{matrix}\text{A cubus}\\ +\text{B plano ter in A.}\end{matrix}\right\}$ æquetur z ſolido bis.

$L\left\{\begin{matrix}\text{B plano-plano-plani}\\ +\text{Z ſolido ſolido.}\\ +\text{Z ſolido}\end{matrix}\right\}$ æquetur G cubo.

$\dfrac{\text{G quadratum} - \text{B plano}}{\text{G}}$ æquabitur A.

Si 1 C + 81 N. *æquetur* 702.
quoniam 378. *plus* 351 *eſt* 729. *Cubus à Latere* 9. *ideo* 81 *ſeu* 6.
eſt 1 N. *de qua quæritur.* $\frac{-27}{9}$

Conſectarium.

E duobus antedictis conſectariis.

Denique ſunt duo latera vnum idemque minus D, alterum idemque maius G, quorum differentia eſt A de qua quæritur.

Itaque L v c. $L\left\{\begin{matrix}\text{B plano-plano plani}\\ +\text{Z ſolido-ſolido.}\end{matrix}\right.$ + Z ſolido.

— L v c. $L\left\{\begin{matrix}\text{B plano-plano-plani}\\ +\text{Z ſolido-ſolido}\end{matrix}\right.$ — Z ſolido.

Eſt A quæſita.

Si 1 C + 6 N. *æquetur* 2.
L c 4 — *L c* 2. *eſt* 1 N *de qua quæritur.*

PROBLEMA II.

CVbum adfectum ſub Latere negatè, ad planum ſub Radice ſolida negatum de Quadrato, reducere.

Oportet autem in æquatione propoſitâ, cubum e triente coefficientis adfectionis, cedere ſolidi comparationis ſub quadruplo quadrato.

Proponatur $\left.\begin{matrix}\text{A cubus}\\ -\text{B plano ter in A}\end{matrix}\right\}$ æquari z ſolido bis.

Oportet facere quod propoſitum eſt.

$\left.\begin{matrix}\text{A in E.}\\ -\text{E quadrato}\end{matrix}\right\}$ æquetur B plano.

Vnde B planum ex huiuſmodi æquationis conſtitutione, intelligitur rectangulum ſub duobus lateribus, quorum maius minuſ-

eâ tranſmutandi doctrina. Statuitor coefficiens vnitas, vſus igitur impendij vel ex eo liquet, quod poteſtates adfectæ ſub vnitate & gradu quocunque, (ſi modo Radix eſt numerus.) non aliter reſoluuntur ac ſi eſſent puræ. neque enim negotium conturbat habenda alioqui (ſed quæ ex iſthoc opere, ſalua fit.) Parabolarum quarum quæque vnitas eſt, ratio.

Et ſi Radix inuenta non eſt numerus, ipſam Radicem de qua quæritur non eſſe numerum ſtatim conuincitur.

1C + 1N. æquatur 10. quoniam proximus cubus eſt 8. cuius Radix eſt 2, quæ ducta in vnitatem & adiuncta 8. facit 10. ideo Radix quæſita eſt 2.

Aequè 1C − 1N æquetur 24. quoniam proxime minor cubus eſt 8, cuius Radix eſt 2. & adſcita vnitate 3. ſub qua & vnitate facto ſolido, quum multatur cubus ex 3. relinquitur 24. ideo Radix quæſita eſt 3.

Proponatur autem 1C. + 1N. æquari 9. quoniam proximus cubus eſt 8, cuius Radix eſt 2. qua ducta in vnitatem & adiuncta 8, facit 10. non etiam 9. ideo 1N eſt Radix irrationalis.

Æque 1C − 1N. æquetur 25. quoniam proxime minor cubus eſt 8. cuius Radix eſt 2. & adſcita vnitate 3. ſub quâ & vnitate facto ſolido quum multatur cubus ex 3. relinquitur 24, non etiam 25. ideo 1N eſt irrationalis.

Oportet autem ad huiuſmodi inſtituendæ tranſmutationis opus, coefficientem quæ imperatur coefficienti æquationis propoſitæ eſſe congenerem, & ſi quidem Radix æquationis propoſitæ eſt eiuſdem quoque generis, concedetur eſſe vt coefficiens propoſitæ æquationis, ad coefficientem imperatam, ita Radix de qua quærebatur, ad nouam ſtatuendam Radicem.

Sin coefficiens gradui Radicis genere communicet, concedetur eſſe vt coefficiens propoſitæ æquationis, ad coefficientem imperatam, ita gradus æque-altus Radicis de qua quærebatur, ad gradum æque altum nouæ ſtatuendæ Radicis.

Et per reſolutionem conceſſi Analogiſmi, exhibebitur ſub nouâ ſpecie valor Radicis quæſitæ, & ſua propoſita æqualitas dirigetur, & ordinabitur noua.

Quod, vt vno aut altero exemplo fiat apertius,

Proponatur $\left.\begin{matrix} A \text{ cubus} \\ + B \text{ in } A \text{ quad} \end{matrix}\right\}$ æquari z ſolido.

Placeat autem æquationem ita tranſmutare, vt adfectio maneat quidem ſub quadrato, ipſaque adfirmetur, ſed coefficiens ſit X non etiam B. Eſto vt B ad X, ita A ad E. ergo $\frac{B \text{ in } E}{X}$ erit A

Quare ſecundum ea quæ proponuntur

$$\left.\begin{matrix} \frac{B \text{ cubus in } E \text{ cubum}}{X \text{ cubo}} \\ + \frac{B \text{ cubo in } E \text{ quad.}}{X \text{ quad.}} \end{matrix}\right\} \text{æquabitur } z \text{ ſolido.}$$

Et omnibus per X cubum ductis, & B cubum diuiſis.

$$\left.\begin{matrix} E \text{ cubus} \\ + X \text{ in } E \text{ quadratum} \end{matrix}\right\} \text{æquabitur } \frac{X \text{ cubo in } z \text{ ſolidum}}{B \text{ cubo.}}$$

Ipſum igitur factum eſt quod oportuit.

Proponatur 1 C. + 20 Q *æquari* 96,000.

1 C + 1 Q *æquabitur* 12. *& fit* 1 N. 2.

Vnde fit Radix primum quæſita 40.

Aliud.

Proponatur $\left.\begin{matrix} A \text{ cubus} \\ - B \text{ quad in } A \end{matrix}\right\}$ æquari z ſolido.

Placeat autem æquationem ita tranſmutare, vt affectio quidem maneat ſub latere, ipſaque negetur, ſed coefficiens ſit X quadratum, non etiam B quadratum. Eſto vt B quad. ad X quad. ita A quad. ad E quad. & conſequenter vt B ad X ita A ad E. ergo

$\frac{B \text{ in } E}{X}$ erit A

Quare ſecundum ea quæ proponuntur

$$\left.\begin{matrix} \frac{B \text{ cubus in } E \text{ cubum}}{X \text{ cubo}} \\ - \frac{B \text{ cubo in } E}{X} \end{matrix}\right\} \text{æquabitur } z \text{ ſolido.}$$

Et omnibus per X cubum ductis, & B cubum diuiſis.

$$\left.\begin{matrix} E \text{ cubus} \\ - X \text{ quadrato in } E \end{matrix}\right\} \text{æquabitur } \frac{X \text{ cubo in } z \text{ ſolidum}}{B \text{ c.}}$$

Ipſum igitur factum eſt quod oportuit.

Proponatur 1 C — 144 N. *æquari* 10, 368.

1 C — 1 N. *æquabitur* 6. *& eſt* 1 N 2.

Vnde fit Radix primum quæſita 4.

Neque vero opus aliter fit in præſcriptis comparationum homogeneis, conceditur nimirùm eſſe, vt magnitudo æqualis poteſtati quæ proponitur adfectæ, ad magnitudinem præſcriptam æque-altam & homogeneam, ita Poteſtas Radicis de quâ quærebạ-

tur, ad Poteſtatem nouæ ſtatuendæ Radicis : & per reſolutionem conceſſi Analogiſmi, exhibetur ſub noua ſpecie valor Radicis quæſitæ, & propoſita æqualitas dirigetur, & ordinabitur noua. Exempli cauſa:

Proponatur $\left.\begin{array}{l}\text{A cubus}\\ +\text{ 3 plano in A}\end{array}\right\}$ æquari z cubo.

Placeat autem æquationem ita tranſmutare, vt poteſtas adfirmate adfecta ſub Radice & coefficiente plano, comparetur D cubo. conceditor eſſe vt z ad D, ita A ad E. ergo $\frac{\text{Z in E}}{\text{D}}$ erit A.

Quare $\left.\begin{array}{l}\frac{\text{Z cubus in E cubum}}{\text{D cubo.}}\\ -\text{ B }\frac{\text{plano in z}}{\text{D}}\text{ in E}\end{array}\right\}$ æquabitur z cubo.

Et omnibus in D cubum ductis, & per z cubum diuiſis.

E cubus + $\frac{\text{B plano in D quad in E}}{\text{Z quad}}$ æquabitur D cubo.

Ipſum igitur factum eſt quod oportuit.

Proponatur 1 C. + 860 N. *æquari* 1,728.

1 C. + 215 N. *æquabitur* 216. *&* *eſt* 1. N. 1.

Vnde ſit Radix primum quæſita. 2.

Anomala Æquationum aliquot Cubicarum ad Quadraticas aut etiam ſimpliciores Reductio.

CAP. IX.

Præparandarum igitur æquationum ſolennes modi ita ſe habent. De irregularibus autem non ſtatuuntur præcepta, quoniam Anomalia illa non magis finita eſt quam Artificis in indagando vis & ſolertia. ad excitandum tamen eam vim & ſolertiam, oportunum eſt ſingularia aliquot æquationum conſtitutiua & reductiua Theoremata adnotaſſe, inſignem aliquam Emphaſin vel elegantiam præ ſe ferentia; qualia iam ſequuntur.

PROPOSITIO I.

Si $\left.\begin{array}{l}\text{A cubus}\\ -\text{ B quadrato bis in A}\end{array}\right\}$ æquetur B cubo.

$\left.\begin{array}{l}\text{A quadratum}\\ -\text{ B in A}\end{array}\right\}$ æquabitur B quadrato.

Ex ijs enim quæ proponuntur manifeſtum fit per Antitheſin, A cubum æquari B cubo + B quadrato bis in A. & addendo vtriq; parti B cubum

A cubum + B cubo } æquari { B cubo bis + B quadrato bis in A.

Omnia applicentur ad A + B, illic oritur

A quad.
— B in A
+ B quadrato.

hic B quad. bis. & conſequenter abiecto vtrinque B quadrato.

A quad. — B in A } æquabitur B quadrato.

Si 1 C. — 18 N. *æquetur* 27. *igitur*
1 Q — 3 N *æquabitur* 9.

PROPOSITIO II.

Si B quad bis in A — A cubo } æquetur B cubo.

A quadratum + B in A } æquabitur B quadrato.

Ex ijs enim quæ proponuntur manifeſtum fit per Antitheſin, A cubum æquari B quad. bis in A — B cubo & auferẽdo vtriq; parti B cubum.

A cubum — B cubo } æquari B quadrato bis in A. — B cubo bis

Omnia applicentur ad A — B. illic oritur

A quad.
+ B in A
+ B quad.

hic B quadr. bis. & conſequenter abiecto vtrinque B quadrato.

A quad. + B in A } æquabitur B quadrato.

Si 18 N. — 1 C. *æquetur* 27. *igitur*
1 Q — 3 N. *æquabitur* 9.

PROPOSITIO III.

Si A cubus — B quad. ter in A } æquetur B cubo bis.

B dupla eſt ipſa A de qua quæritur.

Quoniam enim B dupla, eſt ipſa A de qua quæritur, ergo ex ijs quæ proponuntur B cubus 8 — B quad. in B 6. } æquabitur B cubo bis, quod quidem ita ſe habet.

1 C. — 12 N. *æquetur* 16. *fit* 1 N 4.

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PROPOSITIO IIII.

Si $\left.\begin{array}{l}\text{B quadratum ter in A}\\ -\text{A cubo}\end{array}\right\}$ æquetur B cubo bis.

B eſt ipſa A de qua quæritur.

Quoniam enim B eſt ipſa A de quâ quæritur, ergo ex ijs quæ proponuntur

$\left.\begin{array}{l}\text{B quad. in B ter}\\ -\text{B cubo}\end{array}\right\}$ æquabitur B cubo bis. quod quidem ita ſe habet.

6N — 1C *æquatur l.* 32. *fit* 1 N 2.

PROPOSITIO V.

Si $\left.\begin{array}{l}\text{A cubus}\\ -\text{B in A quad.}\\ +\text{D plano in A}\end{array}\right\}$ æquet. B in D plan. ipſa B eſt A de qua quæritur.

Quoniam enim B eſt A de qua quæritur: ergo ex ijs quæ proponuntur $\left.\begin{array}{l}\text{B cubus}\\ -\text{B in B quadrat.}\\ +\text{D plano in B}\end{array}\right\}$ æquab. B in D plan. quod quidem manifeſto ita ſe habet.

1C — 4Q. + 5N *æquatur* 20. *facto ex* 4 *in* 5. *ergo* 1N *eſt* 4.

PROPOSITIO. VI.

Si $\left.\begin{array}{l}\text{A cubus}\\ +\text{B in A quad}\\ -\text{D quad. in A}\end{array}\right\}$ æquetur B in D quadratum, ipſa D eſt A de qua quæritur.

Quoniam enim ipſa D eſt A de qua quæritur, ergo ex ijs quæ proponuntur, $\left.\begin{array}{l}\text{D cubus}\\ +\text{B in D quad.}\\ -\text{D quad. in D.}\end{array}\right\}$ æquabitur B in D quadratum.

Quod quidem manifeſto ita ſe habet.

1C. + 5Q — 4N. *æquatur* 20. *facto ex* 5. *in* 4. *ergo* 1N. *fit l.* 4.

PROPOSITIO VII.

Si $\left.\begin{array}{l}\text{B in A quad.}\\ +\text{D quad. in A}\\ -\text{A cubo}\end{array}\right\}$ æquetur D quadratio in B, ipſa B vel D, eſt A de qua quæritur.

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Quoniam enim ipſa B eſt A de qua quæritur, ergo ex ijs quæ

proponuntur $\left.\begin{array}{l} \text{B cubus} \\ + \text{D quad in B} \\ - \text{B cubo} \end{array}\right\}$ æquabitur B in D quadratum.

Quod quidem ita manifeſto ſe habet.

Rurſus quoniam ipſa D eſt A de qua quæritur, ergo ex ijs quæ

proponuntur, $\left.\begin{array}{l} \text{B in D quad.} \\ + \text{D cubo} \\ - \text{D cubo} \end{array}\right\}$ æquabitur D quadrato in B.

Quod quidem ita quoque manifeſto ſe habet.

6 Q. + 4 N. — 1 C. æquatur 24. fit 1 N. 6. vel 2.

PROPOSITIO VIII.

Si $\left.\begin{array}{l} \text{D in A quad.} \\ + \text{B in D in A} \\ - \text{A cubo} \end{array}\right\}$ æquetur B cubo $\left.\begin{array}{l} \text{D in A} \\ + \text{B in A} \\ - \text{A quad.} \end{array}\right\}$ æquabitur B quadr.

Ex ijs enim quæ proponuntur, manifeſtum fit per Antitheſin.

$\left.\begin{array}{l} \text{B cubum} \\ + \text{A cubo} \end{array}\right\}$ æquari $\left\{\begin{array}{l} \text{D in A quad.} \\ + \text{D in B in A.} \end{array}\right.$

Vtraque pars applicetur ad A + B, ergo

$\left.\begin{array}{l} \text{A quad.} \\ - \text{B in A.} \\ + \text{B quadrato.} \end{array}\right\}$ æquabitur D in A.

Et per Antitheſin, $\left.\begin{array}{l} \text{D in A} \\ + \text{B in A} \\ - \text{A quad.} \end{array}\right\}$ æquabitur B quadrato.

Si 10 Q. + 20 N. — 1 C. æquetur 8. quia latus cubicum 8. ductum in 10. facit 20. Igitur 12 N. — 1 Q. æquabitur 4. & fit 1 N. 6 — l 32. vel 6 + l 32.

PROPOSITIO IX.

Si $\left.\begin{array}{l} \text{A cubus} \\ - \text{D in A quad} \\ + \text{D in B in A.} \end{array}\right\}$ æquetur B cubo.

$\left.\begin{array}{l} \text{D in A} \\ - \text{B in A} \\ - \text{A quad.} \end{array}\right\}$ æquabitur B quadrato.

Ex ijs enim quæ proponuntur manifeſtum fit per Antitheſin, quum A intelligitur maior quam B,

$\left.\begin{array}{l} \text{A cubum} \\ - \text{B cubo} \end{array}\right\}$ æquari $\left\{\begin{array}{l} \text{D in A quad.} \\ - \text{D in A in B} \end{array}\right.$

Vtraque pars æqualitatis applicetur ad A — B.

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Igitur $\left.\begin{array}{l}\text{A quad.}\\ +\text{B in A}\\ +\text{B quadrato}\end{array}\right\}$ æquabitur D in A.

Et per Antitheſin $\left.\begin{array}{l}\text{D in A}\\ -\text{B in A}\\ -\text{A quad}\end{array}\right\}$ æquabitur B quadrato.

Atqui quum B intelligitur maior quam A.

$\left.\begin{array}{l}\text{B cubus}\\ -\text{A cubo}\end{array}\right\}$ æquabitur $\left\{\begin{array}{l}\text{D in A in B}\\ -\text{D in A quad.}\end{array}\right.$

Vtraque pars æqualitatis applicetur ad B — A.

$\left.\begin{array}{l}\text{B quadrat}\\ +\text{A quad.}\\ +\text{B in A}\end{array}\right\}$ æquabitur D in A. vt ante.

Si 1 c — 10 Q. + 20 N. *æquetur* 8. *quia l* c. 8. *ductum in* 10. *facit* 20. *Igitur*, 8 N. — 1 Q. *æquabitur* 4. *& fit* 1 N. 4 — *l* 12. *vel* 4. + *l* 12.

PROPOSITIO X.

Si $\left.\begin{array}{l}\text{A cubus}\\ -\text{B plano ter in A}\end{array}\right\}$ æquetur L. B plano-plano-plani bis.

$\left.\dfrac{\text{L. B plani ter} + \text{L. B plani 1.}}{\text{L. 2}}\right\}$ fit A de qua quæritur.

Quoniam enim $\left.\dfrac{\text{L. B plani ter} + \text{L. B plani 1}}{\text{L. 2}}\right\}$ eſt ipſa A de qua quæritur.

Ideo ex ijs quæ proponuntur

$\left.\begin{array}{l}\dfrac{\text{L. B plano plano-plani 27.} + \text{L. B plano plano-plani 81.} + \text{L. B plano plano-plani 27.} + \text{L. B plano-plano-plani 1.}}{\text{L. 8.}}\\ \dfrac{-\text{L. B plano-plano-plani 27.} - \text{L. B plano plano plani 9.}}{\text{L. 2}}\end{array}\right\}$ æquatur L B plano-plano-plani 2.

Quod quidem ita ſe habet, ſubducendo in prima æqualitatis parte ab æqualibus, æqualia.

1 c — 6 N. *æquatur* 4.
Igitur l 3 + 1. *fit* 1 N.

PROPOSITIO XI.

Si $\left.\begin{array}{l}\text{B planum ter in A}\\ -\text{A cubo}\end{array}\right\}$ æquetur L. B plano-plano-plani bis.

$\left.\dfrac{\text{L. B plani ter} - \text{L. B plani 1.}}{\text{L. 2}}\right\}$ fit A de qua quæritur

Vt apparet, inſequendo veſtigia antecedentis demonſtrationis.

6 N. — 1 C. *æquatur* 4.

Igitur l 3 — 1. *fit* 1 N. *eaque minor, altera eſt* 2.

Similium Reductionum continuatio.

CAP. X.

PROPOSITIO I.

Si A cubus + B in A quad. ter + B plano in A. æquetur B cubo bis — D plano in B.

A quad. + B in A bis æquetur B quadrato bis. — D plano.

Quoniam enim A quad. + B in A bis. æquatur B quadrato bis — D plano.

Ductis igitur omnibus in A.

A cubus + B in A quad. bis æquabitur B quad in A 2. — D plano in A

Et ijſdem ductis in B. B in A quad. + B quad. in A bis æquabitur B cubo bis — D plano in B.

Iungantur ducta æqualia, æqualibus.

A cubus + B in A quad. ter + B quad. in A bis æquabitur B quad. in A bis — D plano in A. + B cubo bis — D plano in B.

Et deleta vtrinque adfectione B quad. in A bis. & ad æqualitatis ordinationem, tranſlata per Antitheſin D plani in A adfectione.

A cubus + B in A quad. ter + D plano in A. æquabitur B cubo bis — D plano in B.

Quod quidem ita ſe habet.

1 C + 30 Q. + 44 N. *æquatur* 1,560.

Igitur 1 Q + 20 N. *æquabitur* 156. *& fit* 1 N. 6.

PROPOSITIO II.

Si A cubus + B in A quad 3. — D plano in A æquetur B cubo bis. + D plano in B.

A quad + B in A bis æquabitur B quad bis + D plano.

Quoniam enim A quad + B in A bis æquatur B quad. bis + D plano.

Ductis igitur omnibus in A.

$$\left.\begin{array}{l}\text{A cubus}\\ -\text{B in A quad. 2.}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{B quad. in A bis}\\ +\text{D plano in A.}\end{array}\right.$$

Et ductis iiſdem in B. $\left.\begin{array}{l}\text{B in A quad.}\\ +\text{B quad in A bis}\end{array}\right\}\text{æquabitur.}\left\}\begin{array}{l}\text{B cubo bis.}\\ +\text{D plano in B.}\end{array}\right.$

Iungantur ducta æqualia æqualibus.

$$\left.\begin{array}{l}\text{A cubus}\\ +\text{B in A quad. ter}\\ +\text{B quad. in A bis}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{B quad. in A bis}\\ +\text{D plano in A.}\\ +\text{B cubo bis.}\\ +\text{D plano in B.}\end{array}\right.$$

Et deleta vtrinque affectione B quadrati in A bis, & ad ordinationem æqualitatis, tranſlata per Antitheſin D plani in A adfectione.

$$\left.\begin{array}{l}\text{A cubus}\\ +\text{B in A quad. ter}\\ -\text{D plano in A.}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{B cubo bis}\\ +\text{D plano in B.}\end{array}\right.$$

Quod quidem ita ſe habet.

1 C + 30 Q. — 24 N. *æquetur* 2,240.

Igitur 1 Q + 20 N. *æquatur* 224. *& fit* 1 N. 8.

PROPOSITIO III.

Si $\left.\begin{array}{l}\text{A cubus}\\ -\text{B in A quad. ter}\\ +\text{D plano in A.}\end{array}\right\}\text{æquetur}\left\{\begin{array}{l}\text{D plano in B.}\\ -\text{B cubo bis.}\end{array}\right.$

Et ſit B quad. ter, maius D plano.

$$\left.\begin{array}{l}\text{B in A bis}\\ -\text{A quad.}\end{array}\right\}\text{æquetur}\left\{\begin{array}{l}\text{D plano.}\\ -\text{B quad. 2.}\end{array}\right.$$

Quoniam enim $\left.\begin{array}{l}\text{B in A 2.}\\ -\text{A quad.}\end{array}\right\}\text{æquatur}\left\{\begin{array}{l}\text{D plano}\\ -\text{B quad 2.}\end{array}\right.$

Ductis omnibus in B — A

$$\left.\begin{array}{l}\text{B quad in A bis}\\ -\text{B in A quad.}\\ -\text{B in A quad. 2.}\\ +\text{A cubo}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{B quad. in A 2.}\\ -\text{D plano in A}\\ -\text{D plano in B.}\\ +\text{B cubo bis}\end{array}\right.$$

Et ordinatâ æqualitate $\left.\begin{array}{l}\text{A cubus}\\ -\text{B in A quad. ter}\\ +\text{D plano in A}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{D plano in B}\\ -\text{B cubo bis.}\end{array}\right.$

Quod quidem ita ſe habet.

1 C — 30 Q. + 236 N. *æquetur* 360.

Igitur 20 N. — 1 Q *æquatur* 36. *& fit* 1 N. 2, *vel* 18.

Iiſdem poſitis, ipſa A fit quoque B, ſiue B triplum quadratum præſtet, ſiue cedat B plano.

Quoniam enim proponitur $\left.\begin{array}{l}\text{A cubus}\\ -\text{B in A quad. 3.}\\ +\text{D plano in A}\end{array}\right\}\text{æquari}\left\{\begin{array}{l}\text{D plano in B}\\ -\text{B cubo bis.}\end{array}\right.$

Ipſa autem A fit quoque B, igitur

$$\left.\begin{array}{l}\text{B cubus}\\ -\text{B cubo ter}\\ +\text{D plano in B}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{D plano in B.}\\ -\text{B cubo bis.}\end{array}\right.$$

Quod quidem ita ſe habet.

1 C. — 30 Q. + 236 N. *æquatur* 360. *& oſtenſa eſt* 1 N. 2. *vel* 18. *eadem quoque eſt* 10.

1 C. — 30 Q. + 264 N. *æquatur* 640. *fit* 1 N. 4. *vel* 16.

Nam 20 N. — 1 Q. *æquatur* 64. *& fit* 1 N. 4. *vel* 16.

PROPOSITIO IIII.

Si $\left.\begin{array}{l}\text{B in A quad. ter}\\ +\text{D plano in A}\\ -\text{A cubo}\end{array}\right\}\text{æquetur}\left\{\begin{array}{l}\text{B cubo 2.}\\ +\text{D plano in B.}\end{array}\right.$

$$\left.\begin{array}{l}\text{A quad.}\\ -\text{B in A bis}\end{array}\right\}\text{æquatur}\left\{\begin{array}{l}\text{B quad.bis}\\ +\text{D plano.}\end{array}\right.$$

Quoniam enim $\left.\begin{array}{l}\text{A quad.}\\ -\text{B in A bis}\end{array}\right\}\text{æquatur}\left\{\begin{array}{l}\text{B quad.bis}\\ +\text{D plano.}\end{array}\right.$

Ductis omnibus in B — A

$$\left.\begin{array}{l}\text{B in A quad.}\\ -\text{B quad.in A bis}\\ -\text{A cubo}\\ +\text{B in A quad.2.}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{B cubo bis}\\ +\text{B in D planum}\\ -\text{B quad.in A 2.}\\ -\text{D plano in A.}\end{array}\right.$$

Et æqualitate ordinatâ

$$\left.\begin{array}{l}\text{B in A quad. ter}\\ +\text{D plano in A}\\ -\text{A cubo.}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{B cubo bis}\\ +\text{D plano in B.}\end{array}\right.$$

Quod quidem ita ſe habet.

30 Q. + 24 N. — 1 C. *æquatur* 2,240.

Igitur 1 Q. — 20 N. *æquatur* 224. *& fit* 1 N. 28.

Iiſdem expoſitis fit A quoque B.

Quoniam enim proponitur $\left.\begin{array}{l}\text{B in A quad.ter}\\ +\text{D plano in A}\\ -\text{A cubo}\end{array}\right\}\text{æquari}\left\{\begin{array}{l}\text{B cubo bis.}\\ +\text{D plano in B.}\end{array}\right.$

Ipſa autem A fit quoque B. igitur

$$\left.\begin{array}{l}\text{B cubus ter}\\ +\text{D plano in B}\\ -\text{B cubo}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{B cubo bis}\\ +\text{D plano in B.}\end{array}\right.$$

Quod quidem ita ſe habet.

30 Q. + 24 N. — 1 C. *æquatur* 2,240. *fit* 1 N. 10.

PROPOSITIO V.

Si $\left.\begin{array}{l}\text{B in A quad 3}\\ -\text{D plano in A}\\ -\text{A cubo.}\end{array}\right\}\text{æquetur}\left\{\begin{array}{l}\text{B cubo bis.}\\ -\text{D plano in B.}\end{array}\right.$

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A quad. —B in A bis } æquabitur { B quad. bis. —D plano.

Quoniam enim A quad. —B in A bis } æquabitur { B quad. bis —D plano.

Ductis omnibus in B — A

B in A quad. / —B quad. in A bis / —A cubo. / +B in A quad. bis } æquabitur { B cubo bis / —D plano in B. / —B quad. in A bis / +D plano in A.

Et æqualitate ordinata B in A quad. ter / —D plano in A / —A cubo } æquabitur { B cubo bis / —D plano in B.

Quod quidem ita ſe habet.

30 Q.—156 N.—1 C. æquatur 440

Igitur 1 Q—20 N. æquabitur 44. & fit 1 N. 22.

Iiſdem poſitis, fit A quoque B.

Quoniam enim proponitur B in A quad. ter / —D plano in A / —A cubo } æquari { B cubo bis / —D plano in B.

Ipſa autem A fit quoque B, igitur

B cubus 3. / —D plano in B / —B cubo } æquabitur B cubo bis / —D plano in B.

Quod quidem ita ſe habet.

30 Q.—156 N.—1 C. æquatur 440. fit 1 N. 10.

PROPOSITIO VI.

Ad Quadrato-quadraticam pertinens.

Si A quadrato-quadratum. / —X in A cubum bis / +X cubo in A 4. } æquetur X quadrato-quadrato 2.

X quadratum in A quad. 2. / —A quad. quad. } æquetur { X quadrato-quad. 4. / —X cubo in L. X quad. 3.

Ergo X quad. quad. 4. / +A quad. quad. / —X quad. in A quad. 2. // X cub. 2. } æquabitur L. X quad. 3.

Et omnibus quadratis, & per X cubo-cubum 4 ductis, adhibitaque congruenter Metatheſi,

X cubo-cubus in A quad 16. / +X quad. in A cubo-cubum 4. / —A quad. quad quad. quad. / —X quad. quad. in A quad. quad. 12 } æquab. X quad. quad. quad. quad. 4.

Quod quidem ita ſe habet. enimuero ſecundum primam æquationem,

X cubus in A 4. — X in A cubum 2 } æquatur { X quad quad. 2 — A quad. quad

Cuius æqualitatis parte vtrauis quadratâ, omnibuſque ritè ordinatis, in eam ipſam æqualitatem inciditur quadrato-cubicam, quæ expoſita eſt.

Itaque A quad. fit X quadratum plus minúſve latere Binomiæ reſiduæ ſeu negatæ, L. X quad. quad. 12 — X quad. 3.

Sit X 1. A 1 N. 1 QQ. — 2 C. + 4 N. *æquatur* 2.

Igitur 2 Q — 1 QQ. *æquabitur* 4 — *l* 12.

& 1 Q. *fit* 1. *plus minuſue latere binominæ negatæ. l* 12. — 3.

Singularium Aliquot Conſtitutionum, ad Æqualitates multipliciter adfectas pertinentium, Collectio.

CAP. XI.

PROPOSITIO I.

Si A quad. + B in A } æquetur B quadrato.

Eſt B dupla longitudo, ſecta in tria proportionalia ſegmenta, quorum primum idemque maius, eſt B, ſecundum A. quo in opere, dicitur B ſecari media & extrema ratione ſemel ſumpta.

PROPOSITIO II.

Si A cubus + B in A quad. + B quad. in A } æquetur B cubo.

Eſt B dupla longitudo, ſecta in quatuor continuè proportionalis ſegmenta, quorum primum idemque maius, eſt B, ſecundum A. quo in opere, dicitur B ſecari media & extrema ratione duplicata.

PROPOSITIO III.

Si A quad. quad. + B in A cub + B quad. in A quad. + B cubo in A. } æquetur B quad. quadrato.

Eſt B dupla longitudo, ſecta in quinque continuè proportionalia ſegmenta, quorum primum idemque maius, eſt B, ſecundum A. quo in opere, dicitur B ſecari media & extrema ratione triplicata.

PROPOSITIO IIII.

Si A quadrato-cubus
$+$ B in A quad.quad.
$+$ B quad. in A cubum
$+$ B cubo in A quad.
$+$ B quad. quad. in A.
} æquetur B quadrato-cubo.

Eſt B dupla longitudo, ſecta in ſex continuè opproportionalia ſegmenta, quorum primum idemque maius eſt B, ſecundum A. quo in opere, dicitur B ſecari media & extrema ratione quadruplicatâ.

PROPOSITIO V.

Si A cubo-cubus
$+$ B in A quadrato-cubum
$+$ B in A qnad.quad.
$+$ B cubo in A cubum
$+$ B quad.quad in A quad.
$+$ B quadrato cubo in A.
} æquetur B cubo-cubo.

Eſt B dupla longitudo, ſecta in ſeptem continuè proportionalia ſegmenta, quorum primum idemque maius, eſt B, ſecundum A. quo in opere, dicitur B ſecari media & extrema ratione quintuplicata.

Earundem Collectio Altera.

CAP. XII.

PROPOSITIO I.

Si A quad.
$+$ B in A
} æquetur B in Z.

Eſt B prima minor inter extremas in ſerie trium porportionalium, aggregatum vero reliquarum duarum eſt Z, & ſit A ſecunda.

PROPOSITIO II.

Si A cubus
$+$ B in A quad.
$+$ B quad. in A
} æquetur B quad. in Z.

Eſt B prima in ſerie quatuor continuè proportionalium, aggregatum verò reliquarum trium Z, & ſit A ſecunda.

PROPOSITIO III.

Si A quad quad.
−+B in A cubum
−+B quad. in A quad.
−+B cubo in A.
} æquetur B cubo in Z.

Eſt B prima in ſerie quinque continuè proportionalium, aggregatum vero reliquarum quatuor eſt Z, & ſit A ſecunda.

PROPOSITIO IIII.

Si A quadrato cubus.
−+B in A quad. quad.
−+B quad. in A cub.
−+B cubo in A quad.
−+B quad. quad. in A
} æquetur B quad. quad. in Z.

Eſt B prima in ſerie ſex continuè proportionalium, aggregatum vero reliquarum quinque eſt Z, & ſit A ſecunda.

PROPOSITIO V.

Si A cubo-cubus.
−+B in A quadrato-cubum.
−+B quad. in A quad. quad.
−+B cubo in A cubum.
−+B quad. quad. in A quad.
−+B quadrato-cubo in A.
} æquetur B quadrato-cubo in Z.

Eſt B prima in ſerie ſeptem continuè proportionalium, aggregatum vero reliquarum ſex eſt Z, & ſit A ſecunda.

Earundem Collectio Tertia.

CAP. XIII.

PROPOSITIO I.

Si B in A
— A quad.
} æquetur B in Z.

Eſt B prima maior inter extremas in ſerie trium proportionalium, differentia vero duarum reliquarum eſt Z. & ſit A ſecunda. poteſt autem eſſe duplex, nam eſt etiam differentia inter primam & ſecundam.

Sed ſi A quad.
— B in A
} æquetur B in Z. eſt B prima minor inter extremas

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differentia vero duarum reliquarum eſt z, & fit A ſimiliter ſecunda.

Sunto proportionales 4. 6. 9.
9 N. — 1 Q. *æquatur* 18. *fit* 1 N. 6. *atque etiam* 3.
Sed ſi 1 Q — 4 N. *æquetur* 12. *fit* 1 N 6.

PROPOSITIO II.

Si A cubus
— B in A quad.
+ B quad. in A } æquetur B quad. in z.

Eſt B prima in ſerie quatuor continuè proportionalium, differentia vero trium reliquarum alternè ſumptarum, eſt z. & fit A ſecunda.

Sunto proportionales continuè 1. 2. 4. 8.
1 C. — 8 Q. + 64 N. *æquatur* 192. *fit* 1 N. 4.
Vel 1 C — 1 Q. + 1 N. *æquatur* 6. *fit* 1 N. 2.

PROPOSITIO. III.

Si B cubus in A
— B quad. in A quad.
+ B in A cubum
— A quad. quad. } æquetur B cubo in z.

Eſt B prima maior in ſerie quinque continuè proportionalium, differentia vero quatuor reliquarum alternè ſumptarum eſt z, & fit A ſecunda maior inter medias: poteſt autem eſſe duplex.

Sed ſi A quad. quad.
— B in A cub.
+ B quad. in A quad.
— B cubo in A. } æquetur B cubo in z.

Eſt B prima minor inter extremas, differentia vero quatuor reliquarum ſumptarum alternè eſt Z, & fit A ſecunda minor inter medias.

Sunto proportionales continuè 1. 2. 4. 8. 16.
4,096 N. — 256 Q. + 16 C — 1 Q Q. *æquatur* 20,480. *fit* 1 N. 8.
Vel 1 Q Q. — 1 C + 1 Q — 1 N *æquatur* 10. *fit* 1 N. 2.

PROPOSITIO IIII.

Si A quadrato-cubus
— B in A quad. quad.
+ B quad. in A cubum
— B cubo in A quad.
+ B quad. quad. in A } æquetur B quad. quad. in z.

Eſt B prima maior in ſerie ſex continuè proportionalium, differentia vero quinque reliquarum alternè ſumptarum eſt z. & fit A ſecunda.

Sunto proportionales continuè 1. 2. 4. 8. 16. 32.

1 QC. — 32 QQ. + 1,024 C. — 32,768 Q. + 1,048,576 N. *æquatur* 11,534,336. *fit* 1 N. 16.

Vel 1 QC. — 1 QQ. + 1 C. — 1 Q. + 1 N. *æquatur* 22. *& fit* 1 N. 2.

Et hæc ſingula ſuam habent ex Zeteſi demonſtrationem: at quæ iam ſequitur collectio, ſua Analytico examini ſubijcienda libere relinquit Theoremata. pertinet autem ad æqualitates de multiplicibus Radicibus mirè explicabiles.

Collectio Quarta.

CAP. XIIII.

PROPOSITIO I.

Si $\left.\begin{matrix} B + D \text{ in } A \\ - A \text{ quad.} \end{matrix}\right\}$ æquetur B in D.

A explicabilis eſt de qualibet illarum duarum B vel D.

3 N. — 1 Q. *æquetur* 2. *fit* 1 N. 1. *vel* 2.

PROPOSTIO. II.

Si $\left.\begin{matrix} A \text{ cubus} \\ - \left\{\begin{matrix} B \\ D \\ G \end{matrix}\right\} \text{ in } A \text{ quad.} \\ + \left\{\begin{matrix} B \text{ in } D \\ B \text{ in } G \\ D \text{ in } G \end{matrix}\right\} \text{ in } A. \end{matrix}\right\}$ æquetur B in D in G.

A explicabilis eſt de qualibet illarum trium B, D, vel G.

1 C. — 6 Q. + 11 N. *æquatur* 6.

Fit 1 N. 1. 2. *vel* 3.

PROPOSITIO. III.

Si $\left\{\begin{matrix} B \text{ in } D \text{ in } G \\ + B \text{ in } D \text{ in } H \\ + B \text{ in } G \text{ in } H \\ + D \text{ in } G \text{ in } H \end{matrix}\right\}$ in A

B in D

— { B in D, B in G, B in H, D in G, D in H, G in H } in A quad. { æquetur B in D in G in H.

+ { B, D, G, H } in A cubum. }

A explicabilis eſt de qualibet illarum quatuor. B, D, G, H.

50 N. — 35 Q. + 16 C. — 1 QQ. æquatur 24.
fit 1 N. 1. 2. 3. *vel* 4.

PROPOSITIO III.

Si A quadrato-cubus.

— { B, D, G, H, K } in A quad. quad.

+ { B in D, B in G, B in H, B in K, D in G, D in H, D in K, G in H, G in K, H in K } in A cubum.

— { B in D in G, B in D in H, B in D in K, B in G in H, B in G in K, B in H in K, D in G in H, D in G in K, D in H in K, G in H in K } in A quad.

+ { B in D in G in H, B in D in G in K, B in D in H in K, B in G in H in K, D in G in H in K } in A

} æquetur B in D in G in H in K.

A explicabilis eſt de qualibet illarum quinque B, D, G, H, K.

1 QC. — 15 QQ. + 85 C. — 225 Q. + 274 N. *æquatur* 120.

Fit 1 N. 1. 2. 3. 4. *vel* 5.

Atque hæc elegans & perpulchræ ſpeculationis Sylloge, tractatui alioquin effuſo, finem aliquem & Coronida tandem imponito.

FINIS.

VITTORIO EMANUELE BIBLIOTECA IN ROMA

APPENDIX,

AB ALEXANDRO ANDERSONO

OPERI SVBNEXA.

QVANDOQVIDEM Theoremate tertio Capitis ſexti, Tractatus prioris de Æquationum Recognitione, Cubicarum æquationum inuerſarum, in quibus homogenea adfectionis eſt ſub latere, conſtitutionem, elegantius & (ſuo more) acutius, in peculiarem Zeteticorum ad Angulares ſectiones pertinentium, ſiluulam, reiecit Autor noſter: quæ quidem an inter alia eius ſcripta hic vſpiam lateat, an adhuc

Trans Indos Eurumque virens, mortalibus oras
Occupet ignotas.

Nobis nondum conſtitit; placuit (ſuadente ſubtiliſſimi iudicij viro & mihi amiciſſimo Renato Bouclier Iuriſconſulto, tum Mathematico peritiſſimo.) hoc admetiri ἐπίμετρον, ex ijs quæ a me ad hanc rem demonſtrata ſunt. quamuis male feriati quidam homines, infrugi & inſulſi, plagij me inſimulare velint, quaſi Vietæ Theoremata, & demonſtrationes, pro meis venditaſſem: quum nihil in hoc genere præter nuda & demonſtrationibus orba Theoremata, libro octauo variorum, & in Reſponſo ad Problema Adriani Romani à Vieta editis, mihi viſum ſit. quin & omnium illarum

Quoniam enim ipſa B eſt A de qua quæritur, ergo ex ijs quæ proponuntur $\left.\begin{matrix}\text{B cubus}\\ +\text{D quad in B}\\ -\text{B cubo}\end{matrix}\right\}$ æquabitur B in D quadratum.

Quod quidem ita manifeſto ſe habet.

Rurſus quoniam ipſa D eſt A de qua quæritur, ergo ex ijs quæ proponuntur, $\left.\begin{matrix}\text{B in D quad.}\\ +\text{D cubo}\\ -\text{D cubo}\end{matrix}\right\}$ æquabitur D quadrato in B.

Quod quidem ita quoque manifeſto ſe habet.

6 Q. + 4 N. — 1 C. æquatur 24. fit 1 N. 6. vel 2.

PROPOSITIO VIII.

Si $\left.\begin{matrix}\text{D in A quad.}\\ +\text{B in D in A}\\ -\text{A cubo}\end{matrix}\right\}$ æquetur B cubo $\left.\begin{matrix}\text{D in A}\\ +\text{B in A}\\ -\text{A quad.}\end{matrix}\right\}$ æquabitur B quadr.

Ex ijs enim quæ proponuntur, manifeſtum fit per Antitheſin.

$\left.\begin{matrix}\text{B cubum}\\ +\text{A cubo}\end{matrix}\right\}$ æquari $\left\{\begin{matrix}\text{D in A quad.}\\ +\text{D in B in A.}\end{matrix}\right.$

Vtraque pars applicetur ad A + B, ergo

$\left.\begin{matrix}\text{A quad.}\\ -\text{B in A.}\\ +\text{B quadrato.}\end{matrix}\right\}$ æquabitur D in A.

Et per Antitheſin, $\left.\begin{matrix}\text{D in A}\\ +\text{B in A}\\ -\text{A quad.}\end{matrix}\right\}$ æquabitur B quadrato.

Si 10 Q. + 20 N. — 1 C. æquetur 8. quia latus cubicum 8. ductum in 10. facit 20. Igitur 12 N. — 1 Q. æquabitur 4. & fit 1 N. 6 — l 32. vel 6 + l 32.

PROPOSITIO IX.

Si $\left.\begin{matrix}\text{A cubus}\\ -\text{D in A quad}\\ +\text{D in B in A.}\end{matrix}\right\}$ æquetur B cubo.

$\left.\begin{matrix}\text{D in A}\\ -\text{B in A}\\ -\text{A quad.}\end{matrix}\right\}$ æquabitur B quadrato.

Ex ijs enim quæ proponuntur manifeſtum fit per Antitheſin, quum A intelligitur maior quam B,

$\left.\begin{matrix}\text{A cubum}\\ -\text{B cubo}\end{matrix}\right\}$ æquari $\left\{\begin{matrix}\text{D in A quad.}\\ -\text{D in A in B}\end{matrix}\right.$

Vtraque pars æqualitatis applicetur ad A — B.

Igitur A quad. + B in A + B quadrato æquabitur D in A.

Et per Antitheſin D in A — B in A — A quad æquabitur B quadrato.

Atqui quum B intelligitur maior quam A.

B cubus — A cubo æquabitur D in A in B — D in A quad.

Vtraque pars æqualitatis applicetur ad B — A.

B quadrat + A quad. + B in A æquabitur D in A. vt ante.

Si 1 c — 10 Q. + 20 N. *æquetur* 8. *quia l* c. 8. *ductum in* 10. *facit* 20.
Igitur, 8 N. — 1 Q. *æquabitur* 4. *& fit* 1 N. 4 — *l* 12. *vel* 4. + *l* 12.

PROPOSITIO X.

Si A cubus — B plano ter in A æquetur L. B plano-plano-plani bis.

L. B plani ter + L. B plani 1. / L. 2 fit A de qua quæritur.

Quoniam enim L. B plani ter + L. B plani 1 / L. 2 eſt ipſa A de qua quæritur.

Ideo ex ijs quæ proponuntur

L. B plano plano-plani 27.
+ L. B plano plano-plani 81.
+ L. B plano-plano-plani 27.
+ L. B plano-plano-plani 1. / L. 8.
— L. B plano-plano-plani 27.
— L. B plano plano plani 9. / L. 2
æquatur L B plano-plano-plani 2.

Quod quidem ita ſe habet, ſubducendo in prima æqualitatis parte ab æqualibus æqualia.

1 c — 6 N. *æquatur* 4.
Igitur l 3 + 1. *fit* 1 N.

PROPOSITIO XI.

Si B planum ter in A — A cubo æquetur L. B plano-plano-plani bis.

L. B plani ter — L. B plani 1. / L. 2 fit A de qua quæritur

Vt apparet, inſequendo veſtigia antecedentis demonſtrationis.

6 N. — 1 C. æquatur 4.

Igitur l 3 — 1. fit 1 N. eaque minor, altera eſt 2.

Similium Reductionum continuatio.

Cap. X.

PROPOSITIO I.

$$\text{Si}\left.\begin{array}{l}\text{A cubus}\\ +\text{B in A quad ter}\\ +\text{B plano in A.}\end{array}\right\}\text{æquetur}\left\{\begin{array}{l}\text{B cubo bis}\\ -\text{D plano in B.}\end{array}\right.$$

$$\left.\begin{array}{l}\text{A quad.}\\ +\text{B in A bis}\end{array}\right\}\text{æquetur}\left\{\begin{array}{l}\text{B quadrato bis.}\\ -\text{D plano.}\end{array}\right.$$

Quoniam enim $\left.\begin{array}{l}\text{A quad.}\\ +\text{B in A bis.}\end{array}\right\}\text{æquatur}\begin{array}{l}\text{B quadrato bis}\\ -\text{D plano.}\end{array}$

Ductis igitur omnibus in A.

$$\left.\begin{array}{l}\text{A cubus}\\ +\text{B in A quad. bis}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{B quad in A 2.}\\ -\text{D plano in A}\end{array}\right.$$

Et iiſdem ductis in B. $\left.\begin{array}{l}\text{B in A quad.}\\ +\text{B quad. in A bis}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{B cubo bis}\\ -\text{D plano in B.}\end{array}\right.$

Iungantur ducta æqualia, æqualibus.

$$\left.\begin{array}{l}\text{A cubus}\\ +\text{B in A quad. ter}\\ +\text{B quad. in A bis}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{B quad. in A bis}\\ -\text{D plano in A.}\\ +\text{B cubo bis}\\ -\text{D ulano in B.}\end{array}\right.$$

Et deleta vtrinque adfectione B quad. in A bis. & ad æqualitatis ordinationem, tranſlatâ per Antitheſin D plani in A adfectione.

$$\left.\begin{array}{l}\text{A cubus}\\ +\text{B in A quad. ter}\\ +\text{D plano in A.}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{B cubo bis}\\ -\text{D plano in B.}\end{array}\right.$$

Quod quidem ita ſe habet.

1 C + 30 Q. + 44 N. *æquatur* 1,560.

Igitur 1 Q + 20 N. *æquabitur* 156. *& fit* 1 N. 6.

PROPOSITIO II.

$$\text{Si}\left.\begin{array}{l}\text{A cubus}\\ +\text{B in A quad 3}\\ -\text{D plano in A}\end{array}\right\}\text{æquetur}\left\{\begin{array}{l}\text{B cubo bis.}\\ +\text{D plano in B.}\end{array}\right.$$

$$\left.\begin{array}{l}\text{A quad}\\ +\text{B in A bis}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{B quad bis}\\ +\text{D plano.}\end{array}\right.$$

Quoniam enim $\left.\begin{array}{l}\text{A quad}\\ +\text{B in A bis}\end{array}\right\}\text{æquatur}\left\{\begin{array}{l}\text{B quad. bis}\\ +\text{D plano.}\end{array}\right.$

Ductis igitur omnibus in A.

$$\left.\begin{array}{l}\text{A cubus}\\ -\text{B in A quad. 2.}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{B quad. in A bis}\\ +\text{D plano in A.}\end{array}\right.$$

Et ductis iiſdem in B. $\left.\begin{array}{l}\text{B in A quad.}\\ +\text{B quad in A bis}\end{array}\right\}\text{æquabitur.}\left\}\begin{array}{l}\text{B cubo bis.}\\ +\text{D plano in B.}\end{array}\right.$

Iungantur ducta æqualia æqualibus.

$$\left.\begin{array}{l}\text{A cubus}\\ +\text{B in A quad. ter}\\ +\text{B quad. in A bis}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{B quad. in A bis}\\ +\text{D plano in A.}\\ +\text{B cubo bis.}\\ +\text{D plano in B.}\end{array}\right.$$

Et deleta vtrinque affectione B quadrati in A bis, & ad ordinationem æqualitatis, tranſlata per Antitheſin D plani in A adfectione.

$$\left.\begin{array}{l}\text{A cubus}\\ +\text{B in A quad. ter}\\ -\text{D plano in A.}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{B cubo bis}\\ +\text{D plano in B.}\end{array}\right.$$

Quod quidem ita ſe habet.

$1C + 30Q - 24N.$ *æquetur* $2,240.$

Igitur $1Q + 20N.$ *æquatur* $224.$ *& fit* $1N.8.$

PROPOSITIO III.

Si $\left.\begin{array}{l}\text{A cubus}\\ -\text{B in A quad. ter}\\ +\text{D plano in A.}\end{array}\right\}\text{æquetur}\left\{\begin{array}{l}\text{D plano in B.}\\ -\text{B cubo bis.}\end{array}\right.$

Et ſit B quad. ter, maius D plano.

$$\left.\begin{array}{l}\text{B in A bis}\\ -\text{A quad.}\end{array}\right\}\text{æquetur}\left\{\begin{array}{l}\text{D plano.}\\ -\text{B quad. 2.}\end{array}\right.$$

Quoniam enim $\left.\begin{array}{l}\text{B in A 2.}\\ -\text{A quad.}\end{array}\right\}\text{æquatur}\left\{\begin{array}{l}\text{D plano}\\ -\text{B quad 2.}\end{array}\right.$

Ductis omnibus in B − A

$$\left.\begin{array}{l}\text{B quad in A bis}\\ -\text{B in A quad.}\\ -\text{B in A quad. 2.}\\ +\text{A cubo}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{B quad. in A 2.}\\ -\text{D plano in A}\\ -\text{D plano in B.}\\ +\text{B cubo bis}\end{array}\right.$$

Et ordinatâ æqualitate $\left.\begin{array}{l}\text{A cubus}\\ -\text{B in A quad. ter}\\ +\text{D plano in A}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{D plano in B}\\ -\text{B cubo bis.}\end{array}\right.$

Quod quidem ita ſe habet.

$1C - 30Q + 236N.$ *æquetur* $360.$

Igitur $20N. - 1Q$ *æquatur* $36.$ *& fit* $1N.2,$ *vel* $18.$

Iiſdem poſitis, ipſa A fit quoque B, ſiue B triplum quadratum præſtet, ſiue cedat B plano.

Quoniam enim proponitur $\left.\begin{array}{l}\text{A cubus}\\ -\text{B in A quad. 3.}\\ +\text{D plano in A}\end{array}\right\}\text{æquari}\left\{\begin{array}{l}\text{D plano in B}\\ -\text{B cubo bis.}\end{array}\right.$

Ipſa autem A fit quoque B, igitur

$$\left.\begin{array}{l}\text{B cubus}\\ -\text{B cubo ter}\\ +\text{D plano in B}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{D plano in B.}\\ -\text{B cubo bis.}\end{array}\right.$$

Quod quidem ita ſe habet.

1 C. — 30 Q. + 236 N. *æquatur* 360. *& oſtenſa eſt* 1 N. 2. *vel* 18. *eadem quoque eſt* 10.

1 C. — 30 Q. + 264 N. *æquatur* 640. *fit* 1 N. 4. *vel* 16.

Nam 20 N. — 1 Q. *æquatur* 64. *& fit* 1 N. 4. *vel* 16.

PROPOSITIO IIII.

Si $\left.\begin{array}{l}\text{B in A quad. ter}\\ +\text{D plano in A}\\ -\text{A cubo}\end{array}\right\}\text{æquetur}\left\{\begin{array}{l}\text{B cubo 2.}\\ +\text{D plano in B.}\end{array}\right.$

$$\left.\begin{array}{l}\text{A quad.}\\ -\text{B in A bis}\end{array}\right\}\text{æquatur}\left\{\begin{array}{l}\text{B quad. bis}\\ +\text{D plano.}\end{array}\right.$$

Quoniam enim $\left.\begin{array}{l}\text{A quad.}\\ -\text{B in A bis}\end{array}\right\}\text{æquatur}\left\{\begin{array}{l}\text{B quad. bis}\\ +\text{D plano.}\end{array}\right.$

Ductis omnibus in B — A

$$\left.\begin{array}{l}\text{B in A quad.}\\ -\text{B quad. in A bis}\\ -\text{A cubo}\\ +\text{B in A quad. 2.}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{B cubo bis}\\ +\text{B in D planum}\\ -\text{B quad. in A 2.}\\ -\text{D plano in A.}\end{array}\right.$$

Et æqualitate ordinatâ

$$\left.\begin{array}{l}\text{B in A quad. ter}\\ +\text{D plano in A}\\ -\text{A cubo.}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{B cubo bis}\\ +\text{D plano in B.}\end{array}\right.$$

Quod quidem ita ſe habet.

30 Q. + 24 N. — 1 C. *æquatur* 2,240.

Igitur 1 Q. — 20 N. *æquatur* 224. *& fit* 1 N. 28.

Iiſdem expoſitis fit A quoque B.

Quoniam enim proponitur $\left.\begin{array}{l}\text{B in A quad. ter}\\ +\text{D plano in A}\\ -\text{A cubo}\end{array}\right\}\text{æquari}\left\{\begin{array}{l}\text{B cubo bis.}\\ +\text{D plano in B.}\end{array}\right.$

Ipſa autem A fit quoque B. igitur

$$\left.\begin{array}{l}\text{B cubus ter}\\ +\text{D plano in B}\\ -\text{B cubo}\end{array}\right\}\text{æquabitur}\left\{\begin{array}{l}\text{B cubo bis}\\ +\text{D plano in B.}\end{array}\right.$$

Quod quidem ita ſe habet.

30 Q. + 24 N. — 1 C. *æquatur* 2,240. *fit* 1 N. 10.

PROPOSITIO V.

Si $\left.\begin{array}{l}\text{B in A quad. 3}\\ -\text{D plano in A}\\ -\text{A cubo.}\end{array}\right\}\text{æquetur}\left\{\begin{array}{l}\text{B cubo bis.}\\ -\text{D plano in B.}\end{array}\right.$

A quad.
—B in A bis } æquabitur { B quad. bis.
—D plano.

Quoniam enim A quad.
—B in A bis } æquabitur { B quad.bis
—D plano.

Ductis omnibus in B — A

B in A quad.
—B quad.in A bis
—A cubo.
+B in A quad.bis } æquabitur { B cubo bis
—D plano in B.
—B quad.in A bis
+D plano in A.

Et æqualitate ordinata B in A quad.ter
—D plano in A
—A cubo } æquabitur { B cubo bis
—D plano in B.

Quod quidem ita ſe habet.

30 Q.—156 N.—1 C. *æquatur* 440

Igitur 1 Q—20 N. *æquabitur* 44. *& fit* 1 N. 22.

Iiſdem poſitis, fit A quoque B.

Quoniam enim proponitur B in A quad,ter
—D plano in A
—A cubo } æquari { B cubo bis
—D plano in B

Ipſa autem A fit quoque B, igitur

B cubus 3.
—D plano in B
—B cubo } æquabitur B cubo bis
—D plano in B.

Quod quidem ita ſe habet.

30 Q.—156 N.—1 C. *æquatur* 440. *fit* 1 N. 10.

PROPOSITIO VI.

Ad Quadrato-quadraticam pertinens.

Si A quadrato-quadratum.
—X in A cubum bis
+X cubo in A 4. } æquetur X quadrato-quadrato 2.

X quadratum in A quad.2.
—A quad.quad. } æquetur { X quadrato-quad.4.
—X cubo in L. X quad.3.

Ergo X quad.quad. 4.
+A quad.quad.
—X quad.in A quad.2.
————————
X cub.2. } æquabitur L.X quad. 3.

Et omnibus quadratis, & per X cubo-cubum 4 ductis, adhibitaque congruenter Metatheſi,

X cubo-cubus in A quad 16.
+X quad.in A cubo-cubum 4.
—A quad.quad quad.quad.
—X quad.quad.in A quad.quad.12 } æquab. X quad.quad. quad. quad. 4.

Quod quidem ita ſe habet. enimuero ſecundum primam æquationem,

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X cubus in A 4.
— X in A cubum 2 } æquatur { X quad quad. 2
— A quad. quad

Cuius æqualitatis parte vtrauis quadratâ, omnibuſque ritè ordinatis, in eam ipſam æqualitatem inciditur quadrato-cubicam, quæ expoſita eſt.

Itaque A quad. fit X quadratum plus minúſve latere Binomiæ reſiduæ ſeu negatæ, L. X quad. quad. 12 — X quad. 3.

Sit X 1. A 1 N. 1 QQ. — 2 C. + 4 N. *æquatur* 2.

Igitur 2 Q — 1 QQ. *æquabitur* 4 — *l* 12.

& 1 Q. *fit* 1. *plus minuſue latere binominæ negatæ. l* 12. — 3.

Singularium Aliquot Conſtitutionum, ad Æqualitates multipliciter adfectas pertinentium, Collectio.

CAP. XI.

PROPOSITIO I.

Si A quad.
+ B in A } æquetur B quadrato.

Eſt B dupla longitudo, ſecta in tria proportionalia ſegmenta, quorum primum idemque maius, eſt B, ſecundum A. quo in opere, dicitur B ſecari media & extrema ratione ſemel ſumpta.

PROPOSITIO II.

Si A cubus
+ B in A quad.
+ B quad. in A } æquetur B cubo.

Eſt B dupla longitudo, ſecta in quatuor continuè proportionalis ſegmenta, quorum primum idemque maius, eſt B, ſecundum A. quo in opere, dicitur B ſecari media & extrema ratione duplicata.

PROPOSITIO III.

Si A quad. quad.
+ B in A cub
+ B quad. in A quad.
+ B cubo in A. } æquetur B quad. quadrato.

Eſt B dupla longitudo, ſecta in quinque continuè proportionalia ſegmenta, quorum primum idemque maius, eſt B, ſecundum A. quo in opere, dicitur B ſecari media & extrema ratione triplicata.

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PROPOSITIO IIII.

Si A quadrato-cubus
$-$+ B in A quad. quad.
$-$+ B quad. in A cubum
$-$+ B cubo in A quad.
$-$+ B quad. quad. in A. } æquetur B quadrato-cubo.

Eſt B dupla longitudo, ſecta in ſex continuè opproportionalia ſegmenta, quorum primum idemque maius eſt B, ſecundum A. quo in opere, dicitur B ſecari media & extrema ratione quadruplicatâ.

PROPOSITIO V.

Si A cubo-cubus
$-$+ B in A quadrato-cubum
$-$+ B in A quad. quad.
$-$+ B cubo in A cubum
$-$+ B quad. quad in A quad.
$-$+ B quadrato cubo in A. } æquetur B cubo-cubo.

Eſt B dupla longitudo, ſecta in ſeptem continuè proportionalia ſegmenta, quorum primum idemque maius, eſt B, ſecundum A. quo in opere, dicitur B ſecari media & extrema ratione quintuplicata.

Earundem Collectio Altera.

CAP. XII.

PROPOSITIO I.

Si A quad.
$-$+ B in A } æquetur B in Z.

Eſt B prima minor inter extremas in ſerie trium porportionalium, aggregatum vero reliquarum duarum eſt Z, & fit A ſecunda.

PROPOSITIO II.

Si A cubus
$-$+ B in A quad.
$-$+ B quad. in A } æquetur B quad. in Z.

Eſt B prima in ſerie quatuor continuè proportionalium, aggregatum verò reliquarum trium Z, & fit A ſecunda.

PROPOSITIO III.

Si A quad quad.
−+B in A cubum
−+B quad.in A quad.
−+B cubo in A.
} æquetur B cubo in Z.

Eſt B prima in ſerie quinque continuè proportionalium, aggregatum vero reliquarum quatuor eſt Z, & fit A ſecunda.

PROPOSITIO IIII.

Si A quadrato cubus.
−+B in A quad.quad.
−+B quad.in A cub.
−+B cubo in A quad.
−+B quad.quad.in A
} æquetur B quad, quad. in Z.

Eſt B prima in ſerie ſex continuè proportionalium, aggregatum vero reliquarum quinque eſt Z, & fit A ſecunda.

PROPOSITIO V.

Si A cubo-cubus.
−+B in A quadrato-cubum.
−+B quad.in A quad.quad.
−+B cubo in A cubum.
−+B quad.quad.in A quad.
−+B quadrato-cubo in A.
} æquetur B quadrato-cubo in Z.

Eſt B prima in ſerie ſeptem continuè proportionalium, aggregatum vero reliquarum ſex eſt Z, & fit A ſecunda.

Earundem Collectio Tertia.

CAP. XIII.

PROPOSITIO I.

Si B in A —A quad. } æquetur B in Z.

Eſt B prima maior inter extremas in ſerie trium proportionalium, differentia vero duarum reliquarum eſt Z. & fit A ſecunda. poteſt autem eſſe duplex, nam eſt etiam differentia inter primam & ſecundam.

Sed ſi A quad. —B in A } æquetur B in Z. eſt B prima minor inter extremas

differentia vero duarum reliquarum eſt z, & fit A ſimiliter ſecunda.

Sunto proportionales 4. 6. 9.
9 N. — 1 Q. *æquatur* 18. *fit* 1 N. 6. *atque etiam* 3.
Sed ſi 1 Q — 4 N. *æquetur* 12. *fit* 1 N 6.

PROPOSITIO II.

Si A cubus
— B in A quad.
+ B quad. in A
} æquetur B quad. in z.

Eſt B prima in ſerie quatuor continuè proportionalium, differentia vero trium reliquarum alternè ſumptarum, eſt z. & fit A ſecunda.

Sunto proportionales continuè 1. 2. 4. 8.
1 C. — 8 Q. + 64 N. *æquatur* 192. *fit* 1 N. 4.
Vel 1 C — 1 Q. + 1 N. *æquatur* 6. *fit* 1 N. 2.

PROPOSITIO. III.

Si B cubus in A
— B quad. in A quad.
+ B in A cubum
— A quad. quad.
} æquetur B cubo in z.

Eſt B prima maior in ſerie quinque continuè proportionalium, differentia vero quatuor reliquarum alternè ſumptarum eſt z, & fit A ſecunda maior inter medias: poteſt autem eſſe duplex.

Sed ſi A quad. quad.
— B in A cub.
+ B quad. in A quad.
— B cubo in A.
} æquetur B cubo in z.

Eſt B prima minor inter extremas, differentia vero quatuor reliquarum ſumptarum alternè eſt Z, & fit A ſecunda minor inter medias.

Sunto proportionales continuè 1. 2. 4. 8. 16.
4,096 N. — 256 Q. + 16 C — 1 Q Q. *æquatur* 20,480. *fit* 1 N. 8.
Vel 1 Q Q. — 1 C + 1 Q — 1 N *æquatur* 10. *fit* 1 N. 2.

PROPOSITIO IIII.

Si A quadrato-cubus
— B in A quad. quad.
+ B quad. in A cubum
— B cubo in A quad.
+ B quad. quad. in A
} æquetur B quad. quad. in z.

Eſt B prima maior in ſerie ſex continuè proportionalium, differentia vero quinque reliquarum alternè ſumptarum eſt z. & fit A ſecunda.

Sunto proportionales continuè 1. 2. 4. 8. 16. 32.

1 QC. — 32 QQ. + 1,024 C. — 32,768 Q. + 1,048,576 N. *æquatur* 11,534,336. *fit* 1 N. 16.

Vel 1 QC. — 1 QQ. + 1 C. — 1 Q. + 1 N. *æquatur* 22. *& fit* 1 N 2.

Et hæc ſingula ſuam habent ex Zeteſi demonſtrationem: at quæ iam ſequitur collectio, ſua Analytico examini ſubijcienda libere relinquit Theoremata. pertinet autem ad æqualitates de multiplicibus Radicibus mirè explicabiles.

Collectio Quarta.

Cap. XIIII.

PROPOSITIO I.

Si B + D in A — A quad. } æquetur B in D.

A explicabilis eſt de qualibet illarum duarum B vel D.

3 N. — 1 Q. *æquetur* 2. *fit* 1 N. 1. *vel* 2.

PROPOSTIO. II.

Si A cubus
— { B, D, G } in A quad.
+ { B in D, B in G, D in G } in A.
} æquetur B in D in G.

A explicabilis eſt de qualibet illarum trium B, D, vel G.

1 C. — 6 Q. + 11 N. *æquatur* 6.
Fit 1 N. 1. 2. *vel* 3.

PROPOSITIO. III.

Si { B in D in G, + B in D in H, + B in G in H, + D in G in H } in A

B in D

$$\left.\begin{array}{l} -\left\{\begin{array}{l}\text{B in D}\\ \text{B in G}\\ \text{B in H}\\ \text{D in G}\\ \text{D in H}\\ \text{G in H}\end{array}\right\}\text{ in A quad.}\\ +\left\{\begin{array}{l}\text{B}\\ \text{D}\\ \text{G}\\ \text{H}\end{array}\right\}\text{ in A cubum.}\end{array}\right\}\text{ æquetur B in D in G in H.}$$

A explicabilis eſt de qualibet illarum quatuor. B, D, G, H.

50 N. — 35 Q. + 16 C. — 1 QQ. *æquatur* 24.

ſit 1 N. 1. 2. 3. *vel* 4.

PROPOSITIO III.

$$\left.\begin{array}{l} \text{Si A quadrato-cubus,}\\ -\left\{\begin{array}{l}\text{B}\\ \text{D}\\ \text{G}\\ \text{H}\\ \text{K}\end{array}\right\}\text{ in A quad.quad.}\\ +\left\{\begin{array}{l}\text{B in D}\\ \text{B in G}\\ \text{B in H}\\ \text{B in K}\\ \text{D in G}\\ \text{D in H}\\ \text{D in K}\\ \text{G in H}\\ \text{G in K}\\ \text{H in K}\end{array}\right\}\text{ in A cubum,}\\ -\left\{\begin{array}{l}\text{B in D in G}\\ \text{B in D in H}\\ \text{B in D in K}\\ \text{B in G in H}\\ \text{B in G in K}\\ \text{B in H in K}\\ \text{D in G in H}\\ \text{D in G in K}\\ \text{D in H in K}\\ \text{G in H in K}\end{array}\right\}\text{ in A quad.}\\ +\left\{\begin{array}{l}\text{B in D in G in H}\\ \text{B in D in G in K}\\ \text{B in D in H in K}\\ \text{B in G in H in K}\\ \text{D in G in H in K}\end{array}\right\}\text{ in A}\end{array}\right\}\text{ æquetur B in D in G in H in K.}$$

APPENDIX,

AB ALEXANDRO ANDERSONO

OPERI SVBNEXA.

QVANDOQVIDEM Theoremate tertio Capitis ſexti, Tractatus prioris de Æquationum Recognitione, Cubicarum æquationum inuerſarum, in quibus homogenea adfectionis eſt ſub latere, conſtitutionem, elegantius & (ſuo more) acutius, in peculiarem Zeteticorum ad Angulares ſectiones pertinentium, ſiluulam, reiecit Autor noſter: quæ quidem an inter alia eius ſcripta hic vſpiam lateat, an adhuc

Trans Indos Eurumque virens, mortalibus oras
Occupet ignotas.

Nobis nondum conſtitit; placuit (ſuadente ſubtiliſſimi iudicij viro & mihi amiciſſimo Renato Bouclier Iuriſconſulto, tum Mathematico peritiſſimo.) hoc admetiri ἐπίμετρον, ex ijs quæ a me ad hanc rem demonſtrata ſunt, quamuis male feriati quidam homines, infrugi & inſulſi, plagij me inſimulare velint, quaſi Vietæ Theoremata, & demonſtrationes, pro meis venditaſſem: quum nihil in hoc genere præter nuda & demonſtrationibus orba Theoremata, libro octauo variorum, & in Reſponſo ad Problema Adriani Romani à Vieta editis, mihi viſum ſit, quin & omnium illarum

circulationum, & progreſſionum rectarum Circuli circumferentijs in ratione Arithmetica ſubtenſarum, vt firmiſſima ita & præclariſſima fundamenta, Theorematis 4. 5. & 7. mei Tractatus ad Sectiones Angulares, demonſtrata, ex quibus Theorematum à Vieta propoſitorum conſtat veritas, a me excogitata & primum edita ſunt. Loquantur qui Vietâ familiariter vſi ſunt, (qui magno meo diſpendio mihi nunquam notus.) quibuſque aduerſariis ſuis inſcripta ſchediaſmata, communicare ſolitus. Sed talpas iſtos loquaces non moror, qui in re non adeo iam obſcurâ, (niſi Bæoticis fortaſſis iſtiuſmodi ingenijs.) prorſus cæcutientes, ſuſceptos a me ad publica promouenda ſtudia labores, (ne mihi fortaſſis cedere videantur) alij adſcribere malunt, quam a me profectos probando, ad meliora incitare. at viderint iſti Bembices quantum a propoſito ſibi aberrarint ſcopo, dum meas lucubrationes, ſummo, & nunquam ſatis laudando viro Franciſco Vietæ attribuunt, quam inde ampla & opima referam ſpolia:

Dum culpare volunt, ſtulti, in contraria currunt.

Sed ſic nõ vincitis: imbellis animi eſt ex alieni nominis iniuriâ ſibi laudes quærere; quin potius quũ alat æmulatio ingenia, & nunc inuidia nunc admiratio incitationem accendat, agite mecum viri vmbratiles, & ſi quod inde mihi laudis accedit, id vobis detractum putatis, veſtris (ſi quid luce dignum poteſtis) laboribus, id damni reſarcire conemini.

ἐν δὲ πείρᾳ, τέλος διαφαίνεται ὧν τις ἐξοχώτερος γένηται

THEOREMA SYSTATICVM

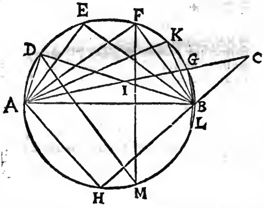
Æqualitatum quarundam Cubicarum adfectarum, in quibus Poteſtati homogenea eſt ſub latere.

SI fuerint duo Triangula rectangula æqualis Hypotenuſæ, & angulus acutus ſubtenſus à Perpendiculo primi, ſit triplus ad angulum acutum ſubtenſum à Perpendiculo ſecundi; Cubus ex dupla Baſe ſecundi, minus ſolido ſub triplo quadrato Hypotenuſæ in eandem Baſin du-

plam ſecundi, æquabitur ſolido ſub quadrato Hypotenuſæ, in duplam Baſin primi.

Rurſus: ſolidum ſub quadrato triplo Hypotenuſæ, in Baſin ſimplam ſecundi, cõtractam protractámve longitudine eius rectæ, quæ poteſt quadrato triplum Perpendiculi eiuſdem, minus eiuſdem Baſis ita contractæ protractæve cubo, æquabitur eidem ſolido ſub quadrato Hypotenúſæ communis, in Baſin duplam primi.

Sit circulus cuius diameter A B, eique inſcribantur vt cunque duo triangula rectangula A F B, A G B, quorum communis hypotenuſa erit ipſa diameter: & ſit angulus F A B, triplus anguli G A B. primi igitur trianguli baſis ſit F A, Perpendiculum F B, (vt in Analyticis conſtitutum eſt.) ſecundi, baſis G A, perpendiculum G B. dico primo; Cubum ex dupla ipſius G A, minus ſolido ſub triplo quadrato ipſius B A, in duplam ipſius G A, æquari ſolido ſub quadrato B A, in duplam ipſius F A. quoniam enim peripheria F B, tripla eſt ipſius G B peripheriæ. Si ſecetur peripheria G F bifariam in K, erunt ſegmenta B G, G K, K F æqualia ducatur iam recta A K. erit igitur ex demonſtratis à nobis Theoremate 5. ad Sectiones Angulares, vt ſemidiameter ad rectam A G, ita recta A G ad compoſitam ex A B, A K: ac proinde vt diameter A B ad duplam ipſius A G, ita A G dupla, ad duplam compoſitam ex A B, A K; igitur A G quadratum quater, æquabitur aggregato quadrati dupli à diametro, & rectanguli ſub diametro A B A K bis.



Iterum: eſt vt A B ad A G bis, ita A K bis, ad duplam compoſitam ex A G, A F, ac proinde aggregatum rectangulorum A B in A G bis, A B in A F bis, æquabitur rectangulo quater ſub A G, A K. & adhibita in priori æqualitate congrua Antitheſi, A G quadratum quater, minus A B quadrato bis, æquabitur A B in AK bis. quare adſumpta communi altitudine A K bis, erit A G quadratum quater, minus A B quadrato bis, ad rectangulum quater ſub A G,

A K. id eſt ad rectangulum ſub A G bis in A K bis, vel ex ſecundæ æqualitatis Analogiſmo, ad aggregatum rectangulorum A B in A G bis A B in A F bis, vt A B ad A G 2. eſt autem & A B quadratum ad A B in A G bis, vt A B ad A G bis: ergo ſi a ſimilibus ſimilia auferantur, nempe ab A G quadrato quater minus A B quadrato bis, ablato A B quadrato, & ab aggregato A B in A G bis, A B in A F bis, ablato A B in A G bis, relinquentur, illic quidem A G quadratum quater minus A B quadrato ter, hic vero A B in A F bis, quæ inter ſe erunt vt A B ad A G bis: quare A G cubus octies, ſiue cubus duplæ ipſius A G, minus A B quadrato ter in A G bis, æquabitur A B quadrato in A F bis: quod erat primo loco demonſtrandum.

Secundo: à puncto B educatur recta B I, ſitque ſegmentum B I inter B punctum, & rectam A G interceptum, æquale ipſius B G duplæ: occurtat autem ipſa B I producta circumferentiæ circuli in D puncto, & ducatur recta A D, quoniam igitur in triangulo rectangulo G B I, latus B I recto ſubtenſum angulo, duplum eſt lateris B G, erit angulus G B I duarum tertiarum vnius recti, ſiue triens duorum rectorum, & latus G I poterit triplum lateris G B, perpendiculi ſcilicet trianguli A G B. dico igitur ſolidum ſub A B quadrato ter in A I, minus cubo ipſius A I, æquari ſolido ſub A B quadrato in A F bis. eſt primum circumferentia G D, angulo G B D ſubtenſa, triens totius circularis peripheriæ, eſt quoque & G B triens peripheriæ B F, quare tota peripheria B D ter, metietur totam peripheriam circularem & præterea ſegmentum B F: metiatur, & ducantur ſubtenſæ B D, D M, M F. quoniam igitur B D, D M æquales ſunt, & eſt A D circumferentia quæ relinquitur ſublatâ B D circumferentia ex ſemicirculo, & M B ea quæ relinquitur ſublata dupla ipſius B D ex integro circulo, eſt igitur M B circumferentia dupla ipſius A D; atqui, ipſi M B circumferentiæ, æqualis eſt circumferentia F D, (æquales ſiquidem ſunt ſubtenſæ B D, D M, ipſis D M, M F.) quare circumferentia F D, dupla erit ipſius D A. ſecetur itaque circumferentia F D, biſariam in E, erunt igitur circumferentiæ A D, D E, E F æquales, & circumferentia A F ipſius A D tripla. ſubtendantur iam rectæ A E, B E. quoniam igitur triangula rectangula G I B, A D I, ſimilia ſunt, erit vt B I ad B G, ita A I ad A D, eſt autem B I dupla ipſius B G, igitur & A I ipſius A D dupla erit. eſt autem ex demonſtratis à nobis Theoremate ſeptimo ad Angulares Sectiones. vt ſemidiameter ad ipſam A D, id eſt vt diameter A B ad ipſam A I, ita A D ad A B minus B E, quare A B quadratum minus A B in B E, æquabitur A I in A D, hoc eſt A I quadrati ſemiſſi: & adhibita congruâ metatheſi, A B quadratum minus A I quadrati ſemiſſe, æquabitur A B in B E. iterum ex iiſdem eſt, vt A B ad A I ita B E ad A F minus A D, rectangulum igitur A B in A F minus A B in A D, æquabitur A I in B E, ergo eadem ad ſumpta altitudine B E, erit vt A B ad A I, ita A B quadratum minus A I quadrati ſemiſſe, ad A I in B E, id eſt ex ſecundæ æqualitatis Analogiſmo, ad A B in A F minus A B in A D: & omnibus duplatis, A B quadratum bis ter minus A I quadrato erit ad A B in A F. bis minus A B in A D bis, vt A B ad A I.

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Eſt quoque vt A B ad A I, ita A B quadratum ad A B in A I, vel A B in A D bis: quare ſi ſimilibus, ſimilia addantur, erit A B quadratum ter minus A I quadrato, ad A B in A F bis, vt A B ad A I; ac proinde A B quadratum ter in A I, minus A I cubo, æquabitur A B quadrato in A F bis: quod erat ſecundo loco oſtendendum.

Tertio: protrahatur ipſa A G quantum ſatis in C, & fiat B C dupla ipſius B G, quare quum rectangulum ſit triangulum B G C, & latus B C recto ſubtenſum angulo, duplum ipſius B G lateris alterius circa rectum, poterit G C quadrato triplum lateris B G, perpendiculi ſcilicet trianguli A G B. dico rurſus, ſolidum triplum ſub A B quadrato in A C, minus Cubo ipſius A C, æquari ſolido ſub A B quadrato & dupla ipſius A F. protrahatur enim C B donec iterum circumferentiam ſecet in H, & ducatur A H. quoniam itaque angulus A B H exterior trianguli A B C, æquátur duobus interioribus oppoſitis C A B, A C B, eſt autem A C B tertia pars vnius recti, ſiue ſemiperipheriæ, (quandoquidem in triangulo rectangulo G C B, latus C B oppoſitum recto, ſtatuitur duplum lateris B G circa rectum.) & angulus B A C tertia pars eſt ipſius anguli B A F, ſeu peripheriæ B F, erit angulus A B H, ſiue peripheria A H tertia pars peripheriæ A H F. & quoniam triangula C H A, C G B ſimilia ſunt, erit C A ad A H, vt C B ad B G, eſt autem C B dupla ipſius B G, quare C A ipſius A H dupla quoque erit. ſecetur iam peripheria H F bifariam in L. & ducatur recta B L. erunt ſegmenta A H, H L, L F æqualia, igitur ex demonſtrati à nobis ſæpius citato Theoremate, erit vt ſemidiameter ad A H, id eſt A B ad A C, ita A H ipſius A C ſemiſſis ad B A minus B L. quare A B quadratum minus A B in B L, æquabitur rectangulo A C in A H, id eſt A C quadrati ſemiſſi. rurſus ex eodem Theoremate, vt A B ad A C, ita B L ad A F minus A H, & rectangulum A B in A F minus A B in A H, æquale erit rectangulo A C in B L. & adhibita congrua metatheſi in priore æqualitate, A B quadratum minus A C quadrati ſemiſſe æquabitur A B in B L. ergo ad ſumpta communi altitudine B L, erit A B quadratum minus A C quadrati ſemiſſe, ad A C in B L, id eſt ex poſterioris æqualitatis Analogiſmo, ad A B in A F minus A B in A H, vt A B ad A C. & omnibus duplatis, erit A B quadratum bis minus A C quadrato, ad A B in A F bis, minus A B in A H bis, vt A B ad A C. eſt quoque A B quadratum, ad A B in A C, vel A B in A H bis, vt A B ad A C, quare ſimilibus, ſi addantur ſimilia, erit A B quadratum ter minus A C quadrato, ad A B in A F bis, vt A B ad A C: ſolidum ergo ſub A B quadrato ter in A C, minus A C cubo, æquale erit ſolido ſub A B quadrato in A F bis: quod erat tertio & vltimo loco oſtendendum.

Atque hinc conſtat lex ab Autore eiuſmodi æqualitatum conſtitutioni appoſita: eſt enim A B diameter, inſcriptarum maxima.

PRIVILEGII SENTENTIA.

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